Some Types of contra τ^*-G-Closed Maps

by

Maysaa Zaki Salman and Dunya Mohamed Hameed

Department of Mathematics , College of Education, Mustansiriyah University , Baghdad - Iraq

Maysaa-zaki@uomustansiriyah.edu.iq and dunya_mahamed@uomustansiriyah.edu.iq

Abstract:

This article introduces new contra closed maps kind called (contra τ^*-G-closed map , contra - (τ^*- G , g) - closed map and contra τ^*-G*-closed map) in topological space and we give the relation among them . Also , several properties of these maps are proved.

Keywords : τ^*-g- closed sets , τ^*-g- open sets , τ^*-g- closed maps and τ^*-g- open maps , contra – closed maps and contra –open maps .

1- Introduction :

In 1982, Dunham .W[4] introduced and study the notion of generalized closure operator CL^* and defined a topology called τ^*-topology. Pushpalatha et al [10] introduced and studied τ^*-generalized closed sets and τ^*-generalized open sets and examined its properties .In [5], Eswaran and Nagaveni .N They are introduced the investigated τ^*-g- closed maps and τ^*-g- open maps .

The notion of contra – closed maps and contra open maps were introduce and study by Baker. C .W,[1]

The aim of this paper is devoted to introduce some kinds of contra τ^*-G^*-closed maps and give the relation between them , also discussion some properties of these maps.

2- Preliminaries:.

Definition (2-1),[4]:

Let Ƀ be subset of X, the generalized closure operate CL^*(Ƀ) is defined by the intersection of all g-closed sets containing Ƀ.

Definition (2-2) ,[10]: Let Ƀ be a subset of space X , then topology $\boldsymbol{\tau}^*$ is defined by $\boldsymbol{\tau}^* = \{G : cL^*(G^c) = G^c\}$.

Definition (2-3):

1- A subset Ƀ of (Ҳ, τ)is called g-closed [8] in Ҳ if CL(Ƀ) $\subseteq$ G whenever Ƀ $\subseteq$ G and G is open in Ҳ. The complement $Ƀ^c$ is g-closed is called (g-open) .

2- A subset Ƀ of (Ҳ, τ^*) is called τ^*-g-closed [10] if CL^*(Ƀ) $\subseteq$ G whenever Ƀ $\subseteq$ G and G is τ^*-open in Ҳ. The complement of τ^*-g-closed set is called (τ^*-g-open).

The class of all g- closed [resp. g-open , τ^*-g-closed-GC(τ^*-GO(Ҳ) Ҳ) d and τ^*-g-open] sets in Ҳ is denoted by GC(Ҳ) [resp. GO(Ҳ) , τ^*-GC(Ҳ) and τ^*-GO(Ҳ)] .

Remark (2-4) , [10] :

(i) all closed (resp. open) set is τ^*-g-closed (resp. τ^*-g- open)set .

(ii) all g-closed (resp. g-open)set is τ^*-g-closed (resp. τ^*-g- open)set .

Definition (2-5): [7] : A space Ҳ is

(a) $T_{1/2}$ - space [7] if all g-closed (resp, g-open) set in Ҳ is closed (resp, open) set

(b) τ^*-Tg space if all τ^*-g-closed set in Ҳ is g-closed in Ҳ.

Definition (2-6):

if k: Ҳ → Ẏ is said to be:

1- Closed (resp. Open) [3] map if for all closed (resp. open) set W in Ҳ , k(W) is a closed(resp. open) set in Ẏ.

2- g-closed map [7] if for all closed set W in Ҳ , k(W) is g- close set in Ẏ.

3- g-open map [7] if for all open set Win Ҳ , k(W) is g-open set in Ẏ .

4- τ^*-G-closed [5] if for all g-closed subset W of Ҳ, then k(W) is τ^*-g-closed subset of Ẏ.

5- τ^*-G-open [5] if for all g-open subset W of Ҳ, then k(W) is τ^*-g-open subset of Ẏ.

6- Contra-Closed [1] if for all closed set W in Ҳ , k(W) is an open set in Ẏ .

7- Contra- Open [1] if for all open set Win Ҳ , k(W) is a closed set in Ẏ.

8- Contra g-closed [2] if for all closed set Win Ҳ , k(W) is g-open set in Ẏ.

3-Some Types of contra τ^*-G-Closed Maps:

Contra closed maps types called [contra τ^*-G-closed , contra - (τ^*- G , g) - closed and contra τ^*-G^*-closed) and the relations between them we will be present in this section.

Definition (3-1):

A map k: Ҳ → Ẏ is said to be **contra τ^*-G –closed** if for all closed set W of Ҳ, then k(W) is τ^*-g- open set of Ẏ.

Example(3-2): Let Ҳ=Ẏ= {ℓ ,υ , $\mathscr{b}$} , τ ={Ҳ,ϕ,{ ℓ ,υ}} and σ ={ Ẏ,ϕ,{ℓ} , { ℓ ,υ}} and let k: (Ҳ,τ)→ (Ẏ, σ) by k(ℓ)= $\mathscr{b}$, k(υ)= ℓ and k($\mathscr{b}$)= υ. It is plain that map k is contra τ^*-G – closed .

Proposition (3-3):

- **(i)** all contra closed map is contra τ^*-G – closed .
- **(ii)** all contra g-closed map is contra τ^*-G – closed .

Proof (i):- Let k: (Ҳ,τ)→ (Ẏ, σ) be contra closed and W is closed set in Ҳ .Thus k(W) is open set in Ẏ , since [all open set is τ^*-g-open] , then k(W) is τ^*-g-open set in Ẏ .Hence k is contra τ^*-G-closed map .

Proof (ii):- Let map k: (Ҳ,τ)→ (Ẏ, σ) be contra g-closed and W is a closed set in Ҳ Thus , k(W) is an g-open set in Ẏ , and since[all g-open set is τ^*-g-open] , then k(W) is τ^*-g-open set in Ẏ .Hence k is contra τ^*-G-closed map .

The proposition (3-3) converse is not true, the next example to show that:

Example (3-4):- Let Ҳ=Ẏ= { ℓ ,υ , $\mathscr{b}$}with the topologies τ ={ Ҳ, ϕ ,{ ℓ} , { ℓ ,υ}} and σ ={ Ẏ,ϕ,{ℓ}}.Let k: (Ҳ,τ)→ (Ẏ, σ) is define by k(ℓ)= ℓ , k(υ)= υ and k($\mathscr{b}$)= $\mathscr{b}$. It is plain that map k is a contra τ^*-G – closed , but k is not contra closed and contra g-closed , since for closed set W={υ,$\mathscr{b}$} in Ҳ , k(W)=k({υ,$\mathscr{b}$ })={ υ,$\mathscr{b}$ } is not open (resp. g-open) in Ẏ .

Next we will define and present some notion and results ,that shall need in this work.

Definition(3-5): A space Ҳ is called **τ^*-$T_{1/2}$- space** if all τ^*-g-closed set in Ҳ is closed in Ҳ.

Example (3-6):-

Let Ҳ= { ℓ ,υ , $\mathcal{b}$ } , τ ={ Ҳ, ϕ ,{ℓ},{υ},{ℓ,υ}}.It to see that Ҳ is τ*-$T_{1/2}$- space, since τ*-GC(Ҳ) ={ Ҳ, ϕ,{$\mathcal{b}$} ,{ℓ,$\mathcal{b}$},{υ,$\mathcal{b}$}}= closed sets in a space Ҳ.

Proposition (3-7): If Ҳ is τ*-$T_{1/2}$- space , then all τ*-g-open set in Ҳ is an open set

Proof :

Let W is τ*-g-open set in Ҳ **,** then W^c is τ*-g – closed in Ҳ ,since Ҳ is τ*-$T_{1/2}$ - space Thus , W^c is closed set in Ҳ , then W is an open in Ҳ .

Same way , we show the next proposition :

Proposition (3-8): If Ҳ is τ*-T_g- space , then all τ*-g-open set in Ҳ is an g-open set

The next proposition give the condition that make converse of proposition (3-3) true:

Proposition (3-9): Let k: (Ҳ,τ)→ (Ẏ, σ) be contra τ*-g - closed map , then k is

(i) Contra closed if Ẏ is τ*-$T_{1/2}$- space .

(ii) Contra g-closed if Ẏ is τ*-T_g- space .

Proof (i): Let a set W be a closed in Ҳ .Since k is contra τ*-g - closed .Thus k(W) is τ*-g - open in Ẏ . Also , since Ẏ is τ*-$T_{1/2}$- space so we get k(W) is an open in Ẏ. Hence, k is contra closed map .

Some way we proof step-ii- .

Definition (3-10):

A map k: Ҳ → Ẏ is said to be **contra -(τ*-G , g)-closed** if for all τ*-g - closed set W of Ҳ, then k(W) is g- open set of Ẏ.

Example(3-11):

Let Ҳ={ ℓ ,υ , $\mathcal{b}$ } , τ ={Ҳ,ϕ,{ℓ,υ}} .It is plain that the identity map k: (Ҳ,τ)→ (Ҳ,τ) is contra -(τ*-G , g)-closed map .

Proposition (3-12):

(i) Every contra -(τ*-G , g) closed map is contra τ*-G – closed .

(ii) Every contra -(τ*-G , g)-closed map is contra g-closed .

Proof (i):-

Let k: (Ҳ,τ)→ (Ẏ, σ) be contra-(τ^*-G, g)- closed map and W is a closed in Ҳ , since [all closed set is τ^*-g – closed] , then W is τ^*-g - closed set in Ҳ Thus , k(W) is g-open set in Ẏ. Also , since [all closed set is g – open set is τ^*-g -open] .Hence , k(W) is τ^*-g -open set in Ẏ .Therefore , k is contra τ^*-G-closed map .

Some way we proof step-ii- .

Is not true the converse of the proposition (3-12), the next examples to show that:

Example (3-13):- Let Ҳ=Ẏ= { ℓ ,𝓋 , 𝒷 } with the topologies τ ={ Ҳ, ϕ ,{ℓ},{𝓋,𝒷}} and σ ={ Ẏ,ϕ,{ℓ}}.Let k: (Ҳ,τ)→ (Ẏ, σ)such that k(ℓ)= ℓ , k(𝓋)= 𝓋 and k(𝒷)= 𝒷. It is plain that a map k is a contra τ^*-G – closed , but k is not contra-(τ^*-G, g)- closed since for τ^*-g - closed set W={𝓋,𝒷} in Ҳ , k(W)=k({𝓋,𝒷 })={ 𝓋,𝒷 } is not g-open set in Ẏ .

Example (3-14):- Let Ҳ=Ẏ= { ℓ ,𝓋 , 𝒷 } with the topologies τ ={ Ҳ, ϕ ,{ℓ}} and σ ={ Ẏ,ϕ,{ℓ}, {𝓋}, {ℓ,𝓋}}.Let k: (Ҳ,τ)→ (Ẏ, σ) such that k(ℓ)= 𝒷 , k(𝓋)= ℓ and k(𝒷)=𝓋. It is plain that a map k is a contra g – closed , but k is not contra-(τ^*-G, g)- closed since for τ^*-g - closed set W={ℓ} in Ҳ , k(W)=k({ℓ})={𝒷} is not g-open set in Ẏ .

Remark(3-15):

The concepts of contra closed maps and contra - (τ^*- G , g) - closed maps are independents. As seen in examples down.

Example (3-16): Let Ҳ= Ẏ = { ℓ ,𝓋 , 𝒷 } with the topologies τ ={ Ҳ, ϕ ,{𝒷},{𝓋,𝒷}} and σ ={Ẏ,ϕ,{ℓ}}.Let k: (Ҳ,τ)→ (Ẏ, σ) is define by k(ℓ)= ℓ , k(𝓋)= 𝒷 and k(𝒷)=𝓋. It is plain that k is contra-(τ^*-G, g)- closed map , but k is not contra closed since for closed set W={ℓ,𝓋} in Ҳ , k(W)=k({ℓ,𝓋})={ℓ,𝒷} is not open in Ẏ .

Example (3-17):- Let Ҳ=Ẏ= { ℓ ,𝓋 , 𝒷 } with the topologies τ ={ Ҳ, ϕ ,{ℓ} , {ℓ,𝒷}} and σ ={ Ẏ,ϕ,{𝓋}, {𝒷}, {𝓋,𝒷}}.Let k: (Ҳ,τ)→ (Ẏ, σ) such that k(ℓ)= ℓ , k(𝓋)= 𝓋 and k(𝒷)=𝒷. It is plain that a map k is a contra closed , but k is not contra-(τ^*-G, g)- closed since for τ^*-g - closed set W={ ℓ,𝓋 } in Ҳ , k(W)=k({ℓ,𝓋 })={ ℓ,𝓋 } is not open set in Ẏ .

The condition that make proposition(3-12) and Remark(3-15) are true, will be present in the next propositions .

Proposition (3-18):

If Let k: (Ӽ,τ)→ (Ẏ, σ) is contra τ^*- G- closed map , Ӽ is τ^*-$T_{1/2}$- space and Ẏ is τ^*-T_g- space **,** then k is contra-(τ^*- G, g) – closed map **.**

Proof : Let W be τ^*-g - closed set in Ӽ .Since Ӽ is τ^*-$T_{1/2}$- space , thus W is a closed set in Ӽ . Also , since k is contra τ^*-G - closed map .Thus k(W) is τ^*-g - open set in Ẏ . Also , by hypotheses Ẏ is τ^*-T_g- space then k(W) is an g-open set in Ẏ. Hence k contra-(τ^*- G, g) – closed .

Same way , we show the next proposition **:**

Proposition (3-19): If k: (Ӽ,τ)→ (Ẏ, σ) is contra closed (resp. g-closed) map and Ӽ is τ^*-$T_{1/2}$- space . then k is contra-(τ^*- G, g) – closed map .

Proposition (3-20): If k: (Ӽ,τ)→ (Ẏ, σ) is contra-(τ^*- G, g) – closed map and Ẏ is $T_{1/2}$- space, then k is contra closed .

Now, we give another types of contra- τ^*-G - closed map is called contra- τ^*-G* - closed map.

Definition (3-21):

A map k: Ӽ → Ẏ is said to be **contra τ^*-G* –closed** if for all τ^*-g -closed set W of Ӽ, then k(W) is open set of Ẏ.

Example(3-22): Let Ӽ=Ẏ= { ℓ ,υ , $\mathscr{b}$ } , τ ={Ӽ,ϕ,{υ,$\mathscr{b}$}} and σ ={ Ẏ,ϕ,{ℓ} ,{υ},{ℓ,υ},{ℓ,$\mathscr{b}$}}.Let k: (Ӽ,τ)→ (Ẏ, σ) define by k(ℓ)= ℓ , k(υ)= υ and k($\mathscr{b}$)= $\mathscr{b}$. It observe k is a contra τ^*-G* – closed .

Proposition (3-23): If k: (Ӽ,τ)→ (Ẏ, σ) is contra τ^*-G* – closed map , then k is

(i) Contra closed map.

(ii) Contra g-closed map .

(iii) Contra τ^*-G – closed map.

(iv) Contra -(τ^*- G, g) – closed map.

IEEE-SEM, Volume 8, Issue 12, December-2020
ISSN 2320-9151

Proof (i): W is a closed set in X .Since [all closed set is τ^*-g-closed), then W is τ^*-g-closed set in X Thus k(W) is an open set in $\dot{Y}$ .Hence k is contra closed map .

Some way we proof step-ii- ,-iii- and –iv-.

The proposition (3-23) converse is not true, the next examples to show that:

Example (3-24): Let $X=\dot{Y}=\{\ell, \mathscr{v}, \mathscr{b}\}$ with the topologies $\tau=\{X, \phi, \{\ell, \mathscr{v}\}\}$ and $\sigma=\{\dot{Y}, \phi, \{\ell\}\}$. Let k: $(X,\tau)\to(\dot{Y},\sigma)$ is define by $k(\ell)=\mathscr{v}$, $k(\mathscr{v})=\mathscr{b}$ and $k(\mathscr{b})=\ell$. It is plain that a map k is a contra closed , but k is not contra-τ^*-G*- closed since for τ^*-g - closed set $W=\{\ell,\mathscr{b}\}$ in X , $k(W)=k(\{\ell,\mathscr{b}\})=\{\ell,\mathscr{b}\}$ is not open in $\dot{Y}$.

Example (3-25):

Let $X=\dot{Y}=\{\ell, \mathscr{v}, \mathscr{b}\}$with the topologies $\tau=\{X, \phi, \{\mathscr{v},\mathscr{b}\}\}$ and $\sigma=\{\dot{Y},\phi,\{\mathscr{v}\},\{\ell,\mathscr{b}\}\}$.Let k: $(X,\tau)\to(\dot{Y},\sigma)$ such that $k(\ell)=\ell$, $k(\mathscr{v})=\mathscr{v}$ and $k(\mathscr{b})=\mathscr{b}$ .It is plain that k is a contra g- closed (resp. contra τ^*-G- closed) map , but k is not contra-τ^*-G*- closed map since for τ^*-g - closed set $W=\{\ell,\mathscr{v}\}$ in X , $k(W)=k(\{\ell,\mathscr{v}\})=\{\ell,\mathscr{v}\}$ is not open set in $\dot{Y}$.

The condition that make proposition(3-23) true, will be present in the next propositions .

Proposition (3-26): A map k: $(X,\tau)\to(\dot{Y},\sigma)$ is contra τ^*- G*- closed map if k is

- **(i)** Contra closed and X is τ^*-$T_{1/2}$- space .
- **(ii)** Contra g- closed and X is τ^*-$T_{1/2}$- space and $\dot{Y}$ is $T_{1/2}$- space .
- **(iii)** Contra τ^*- G- closed map if X ,$\dot{Y}$ are τ^*-$T_{1/2}$- spaces .
- **(iv)** Contra-(τ^*- G, g) – closed map and $\dot{Y}$ is $T_{1/2}$- space.

Proof (i):

Let W be τ^*-g - closed set in X , since X is τ^*-$T_{1/2}$- space so we get , W is a closed set in X .Also , since k is contra closed map Thus , k(W) is an open set in $\dot{Y}$. Therefore , k is contra τ^*- G*- closed .

The proof of steps -ii- ,-iii- and –iv- similar to step-i- .

In the following , will be give some proposition about the composition of these types of contra τ^*-G- closed maps .

Proposition (3-27): Let k: (X,τ)→(Ẏ,σ) be closed map and g: (Ẏ,σ)→(Z,μ) be contra τ^*- G- closed map . Then gok: (X,τ)→ (Z,μ) is contra τ^*- G- closed .

Proof :

Let W be a closed set in X . since k is a closed map .Thus k(W) is closed set in Ẏ . Also , since g is contra τ^*- G- closed map , then g(k(W)) = gok(W) is τ^*-g- open set in Z . Therefore, gok: (X,τ)→ (Z,μ) is a contra τ^*- G- closed .

Similarly , we proof the following corollary.

Corollary(3-28):

Let k: (X,τ)→(Ẏ,σ) and g: (Ẏ,σ)→(Z,μ) be and Ẏ maps . Then gok: (X,τ)→ (Z,μ) is contra τ^*- G- closed if k is closed map and

- **(i)** g is contra closed map .
- **(ii)** g is contra-(τ^*- G, g) – closed map.
- **(iii)** g is contra τ^*- G*- closed map.

Proposition (3-29):

Let k: (X,τ)→(Ẏ,σ) be closed map , g: (Ẏ,σ)→(Z,μ) be contra closed and X is τ^*-$T_{1/2}$- space , then gok: (X,τ)→ (Z,μ) is contra τ^*- G*- closed map.

Proof :

Let W is a τ^*-g-closed set in X .Since X is τ^*- $T_{1/2}$- space , then W is a closed set in X also ,since k is closed map , then k(W) is a closed set in Ẏ. since g is contra closed map Thus , g(k(W)) = gok(W) is an open set in Z. Therefore , gok: (X,τ)→ (Z,μ) is contra τ^*- G*- closed.

Same way , we show the next proposition

Proposition (3-30):

(1) If k: (X,τ)→(Ẏ,σ) is closed map , g: (Ẏ,σ)→(Z,μ) is contra - closed and X is τ^*-$T_{1/2}$- space , then gok: (X,τ)→ (Z,μ) is contra τ^*- G* - closed map.

(2) If k: (X,τ)→(Ẏ,σ) is g- closed map , g: (Ẏ,σ)→(Z,μ) be contra τ^*- G*- closed and X is τ^*-$T_{1/2}$- space , then gok: (X,τ)→ (Z,μ) is contra τ^*- G*- closed map.

(3) If k: (X,τ)→(Ẏ,σ) is closed map , g: (Ẏ,σ)→(Z,μ) is contra g- closed ,and X is τ^*-$T_{1/2}$- space , then gok: (X,τ)→ (Z,μ) is contra (τ^*- G*,g) - closed map.

Proposition (3-31):

Let k: (X,τ)→(Ẏ,σ) be contra closed map and g: (Ẏ,σ)→(Z,μ) be open map then gok: (X,τ)→ (Z,μ) is contra τ^*- G- closed map.

Proof : Let W be a closed set in X ,since k is contra closed map , then k(W) is an open set in Ẏ. Also , since g is open map . Thus , g(k(W)) = gok(W) is an open set in Z and [all pen set is τ^*- g- open] . Hence , gok(W) is τ^*- g- open set in Z .Therefore , gok: (X,τ)→ (Z,μ) is contra τ^*- G*- closed.

Proposition (3-32):

Let k: (X,τ)→(Ẏ,σ) be contra τ^*- G*- closed map and g: (Ẏ,σ)→(Z,μ) be open map then gok: (X,τ)→ (Z,μ) is contra τ^*- G*- closed map.

Proof :

Let W be a τ^*- g-closed set in X ,since k is contra τ^*- G*-closed map , then k(W) is an open set in Ẏ. Also , since g is open map . Thus , g(k(W)) = gok(W) is an open set in Z and .Therefore , gok: (X,τ)→ (Z,μ) is contra τ^*- G*- closed.

Proposition (3-33):

Let k: (X,τ)→(Ẏ,σ) be contra (τ^*- G ,g)- closed map, g: (Ẏ,σ)→(Z,μ) be open map and Ẏ is $T_{1/2}$- space ,then gok: (X,τ)→ (Z,μ) is contra τ^*- G*- closed map.

Proof :

Let W be a τ^*- g-closed set in X ,since k is contra (τ^*- G ,g)-closed map , then k(W) is g-open set in Ẏ , by hypotheses Ẏ is $T_{1/2}$- space , then by Definition(2-5) we get k(W) is an open set in Ẏ , Also , since g is open map . Thus , g(k(W)) = gok(W) is an open set in Z and .Therefore , gok: (X,τ)→ (Z,μ) is contra τ^*- G*- closed.

References:

[1] Baker. C.W; "Contra- open function and contra- closed functions", Math. Today, 15(1997) .19-24.

[2] Balasubramanian .S. ," Almost contra vg-open and almost contra vg-closed mappings " , International Journal of Advanced Scientific and Techical Research.Issue 2 ,Vol(2) ,2015 ,78-92 .

[3] Biswas .N.," On some mapping in topological spaces ", Bull .Call .Math .Soc,61(1969), 127-135.

[4] Dunham, W., "A New Closure Operator For Non-T-Topologies", Kyungpoof Math. J. 22 (1982) 55-60.

[5] Eswaran ,S and Nagaveni .N," τ^*-Generalized homeomorphism in Topological Spaces ", International J. of Computer Applications ,Vol(40) , No.1 , 2012 , 38-42

[6] Eswaran,S. and Pushpalatha, A. "τ^*-Generalized Continuous Maps in Topological Spaces", International J. of Math. Sci. and Eng. Appls. (IJMSEA) ISSN 0973-9424, Vol. 3, No. IV, (2009), pp. 67-76.

[7] Jamhour, M. E., "On G-Closed Mappings", M.Sc. Thesis, Al-Mustansirya University, 2002.

[8] Levien, N., "Generalized Closed Set in Topology", Rend. Circ. Math. Palermo (2) 19, (1970), 89-96.

[9] Malgham, S. R., "Generalized Closed Maps", J. Karnatak Univo 27, (1982), 82-88.

[10] Pushpalatha, A.; Eswaran, S. and Rajarubi, P., " τ^*-Generalized Closed Sets in Topological Spaces", Proceeding of World Congress on Engineering (2009), Vol. (11), W(K), 2009, July 1-3, London, U.K., 1115-1117.