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SEVERAL WORKS ON EXISTING DISTRIBUTIONS

Abstract: The graphs of density function have inspired to take many extensions of it. This paper is the stem from the attracting characteristics of the distribution. This paper studies some more theoretical properties of the density curves.

Key Words: Random variable, arrival rate, probability density function, queuing system.

Introduction This paper is continuous work to the previous ones [2], [18] which are briefed here. The Random variable of interest is to "Stay only n number arrivals in the system in a particular time interval". The arrivals to system stay in the system while during the queue, acquiring the service and while probing for some data in the system. Instead of asking "how many arrivals take place in a particular time interval (Poisson)", we ask for "how likely the system to have n number of arrivals in particular time interval". Since X is continuous, the PDF should be a function. We had made some inferences about this unknown function. This means the probability distribution that takes into account of measurements those we have surveyed for a considerable period of time. So the output of the inference problem is the distributions of X. We charted the histograms for different number of arrivals staying in the system from which we found the density curves.

In which case its probability density function is given by

$$f(x) = \begin{cases} \dfrac{e^{-\lambda x}(\lambda x)^n \lambda}{n!} & when\, x \geq 0 \\ 0 & otherwise \end{cases}$$

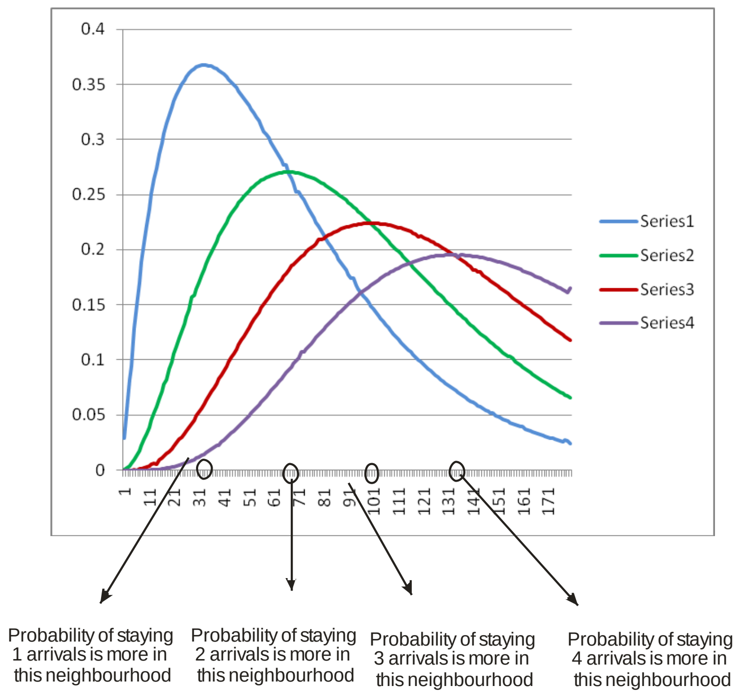

FIGURE - 2

By observing the density function graph, we would note that each series peaks at particular point and then starts to descend. That is each series had peak point in particular time interval.

A short survey conducted in a bank queuing system to get the customer arrival rate.

It is observed that an average of 3 customers arriving for every 100 seconds to a particular bank. i.e. λ = 0.03 cm/sec

For n = 1

First curve got its peak point at t = $\frac{n}{\lambda}$ = $\frac{1}{\lambda}$ = 33 seconds that is in the neighborhood of $\frac{1}{\lambda}$, the curve gets its highest position. We had taken this $\frac{1}{\lambda}$ = 33 seconds.

For n = 2

Consider $n\left(\frac{1}{\lambda}\right)\pm or\ n\left(\frac{1}{\lambda}\right)\pm 2$

$n(33)\pm 1\ or\ n(33)\pm 2$

In particular we take n(33) +1 or n(33) +2

2(33)+1 or 2(33)+2

67 or 68

The 2nd curve got its peak point in the neighborhood of $n\left(\frac{1}{\lambda}\right)$=66 seconds, probably at 67 seconds.

For n = 3:

Consider

$n\left(\frac{1}{\lambda}\right)\pm 1\ or\ \ n\left(\frac{1}{\lambda}\right)\pm 2$

$3(33)1\pm\ or\ 3(33)\pm 2$

In Particular $3(33)+1\ or\ 3(33)+2$

100 or 101

The third curve (n = 3) got its peak point in the neighborhood of $\frac{n}{\lambda}=3(33)=99$ seconds probably at 100 seconds the curve attained its highest position.

For n = 4

Consider $n\left(\frac{1}{\lambda}\right)\pm 1\ \ or\ n\left(\frac{1}{\lambda}\right)\pm 2$

$$4(33)\pm 1 \ or \ 4(33)\pm 2$$

In particular 4(33) +1 or 4 (33) +2

133 or 134

The 4th curve got its peak in the neighborhood of $\frac{n}{\lambda}=\frac{4}{\lambda}=4(33)=132$ seconds.

Probably at 133 seconds curve attained its highest position.

In most of the cases the curves attains its highest position in the neighborhood of $\frac{n}{\lambda}$ and that too probably at $\frac{n}{\lambda}$ +1.

The above calculation generalized and indicated by the formula.

For n = 1:

The curve peaks in the neighborhood of $\frac{n}{\lambda}$, most probably at $\frac{n}{\lambda}$ +1 only.

For n $\geq$ 2:

These curves are attaining their highest in the neighborhood of $\frac{n}{\lambda}$ and most probably at $\frac{n}{\lambda}$ +1

The theoretical proof for the above calculations:

From the above calculations for $n\geq 2$ the curves are attaining their highest position at some neighborhood of $\frac{n}{\lambda}$, and most probably at $\frac{n}{\lambda}$ +1

We prove this by assuming contradiction.

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If possible, let us suppose this point is not highest point. i.e. Probability mass accumulated under that point is not more. So the curve is getting peak point at some time t. After that curve is coming down and runs parallel to x-axis at the end.

$\therefore f\left(\frac{n}{\lambda}+1\right) < f(t)$ Where t is other than $\frac{n}{\lambda}+1$ *and* $n \geq 2$

Where $f(x) = \frac{e^{-\lambda x}(\lambda x)^n \lambda}{n!}$

$$\frac{e^{-\lambda\left(\frac{n}{\lambda}+1\right)}\lambda^n\left(\frac{n}{\lambda}+1\right)^n \lambda}{n!} < \frac{e^{-\lambda t}(\lambda t)^n{}_{\lambda}}{n!}$$

$$e^{-(n+\lambda)}(n+\lambda)^n < e^{-\lambda t}(\lambda t)^n$$

Case 1: When t = 0

$$e^{-(n+\lambda)}(n+\lambda)^n < 0$$

This is absurd because

(1) Exponential gives a positive value.

(2) λ is intensity factor(arrival rate)>0

(3) $n \geq 2$

$\therefore$ L.H.S cannot be less than 0

$\therefore f\left(\frac{n}{\lambda}+1\right)$ is not less than f(t)

Case 2: When $t < \frac{n}{\lambda}+1$

$$e^{-(n+\lambda)}(n+\lambda)^n < e^{-\lambda t}(\lambda t)^n$$

With $e^{-(n+\lambda)}$ multiplied b y $(n+\lambda)^n$ cannot be less than $e^{-(n+\lambda)}(\lambda t)^n$ with $t < \frac{n}{\lambda}+1$

Case 3: When $t > \frac{n}{\lambda}+1$

$e^{-(n+\lambda)}(n+\lambda)^n < e^{-\lambda t}(\lambda t)^n$

Due to exponentials on both sides, L.H.S. cannot be less than R.H.S. with

t > $\frac{n}{\lambda}+1$

$\therefore f\left(\frac{n}{\lambda}+1\right)$ Is not less than f(t) with $t > \frac{n}{\lambda}+1$

$\therefore$ In all cases $\therefore f\left(\frac{n}{\lambda}+1\right)$ is not less than f(t).

$\therefore$ We get in the neighborhood of $\frac{n}{\lambda}$ the maximum point of f(x).

In most of the cases probably at t = $\frac{n}{\lambda}+1$ Curves are attaining their highest.

CONCLUSION

By using the graph of the density function this paper studied some theoretical properties of the density function curves. Using density graphs we developed the relation or the neighborhood of time where curves attain its maximum position in this graph. And the most interesting application is neighborhood of $\frac{n}{\lambda}$ the maximum point of f(x) in this graph. In most of the cases in this probably at t = $\frac{n}{\lambda}+1$ curves are attaining their highest.

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