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MEAN, VARIANCE AND STANDARD DEVIATION TO THE EMPIRICAL DISTRIBUTION IN CONTINUOUS CASE

Abstract: This paper proposes the Mean, Variance and Standard Deviation for density function

$$f(x) = \begin{cases} \frac{e^{-\lambda x}(\lambda x)^n \lambda}{n!} & when\, x \geq 0 \\ 0 & otherwise \end{cases}$$

for the random variable "Staying n number of persons in the system in a particular interval".

Key Words: Random variable, Continuous probability distribution, Arrival rate, Density function, Mean, Variance, Standard Deviation.

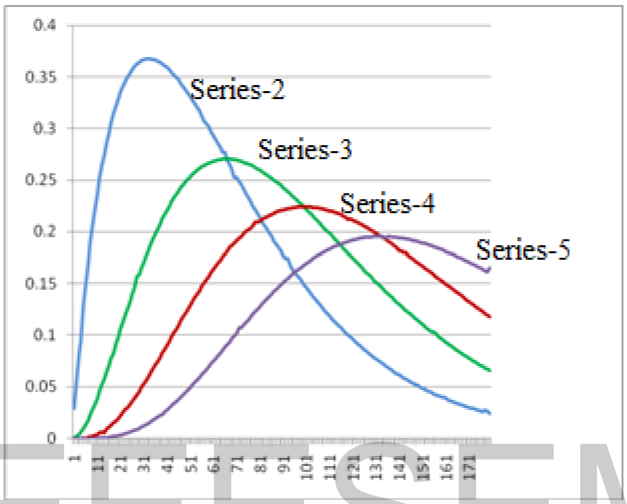
Introduction Statistics has the most interesting solutions for the problems in several fields due to its universality. Several new distributions have been developed by taking some subtle transformations on the existing distributions. This paper is continuous work to the previous one [2] which is briefed here. The Random variable of interest is to "Stay only n number arrivals in the system in a particular time interval". The arrivals to system stay in the system while during the queue, acquiring the service and while probing for some data in the system. Instead of asking "how many arrivals take place in a particular time interval (Poisson)", we ask for "how likely the system to have n number of arrivals in particular time interval". Since X is continuous, the PDF should be a function. We had made some inferences about this unknown function. This means the probability distribution that takes into account of measurements those we have surveyed for a considerable period of time. So the output of the inference problem is the distributions of X. We charted the histograms for different number of arrivals staying in the system from which we found the density curves.

In which case its probability density function is given by

$$f(x) = \begin{cases} \dfrac{e^{-\lambda x} (\lambda x)^n \lambda}{n!} & when\, x \geq 0 \\ 0 & otherwise \end{cases}$$

Graph of density function:

Figure 1



Series 1 is the probability density function when n = 1.

Series 2 is the probability density function when n = 2.

Series 3 is the probability density function when n = 3.

Series 4 is the probability density function when n = 4.

X- axis represents time, Y- represents f(x).

To extend this distribution theory, the interesting properties of all statistical distributions mean, variance and standard deviation are studied which are widely used in several fields insurance, management, business and finance etc.

MEAN, VARIANCE AND STANDARD DEVIATION OF THE PROBABILITY DISTRIBUTION

The probability density function is

$$f(x)=\begin{cases}\dfrac{e^{-\lambda x}(\lambda x)^{n}\lambda}{n!} & when\, x\geq 0\\ 0 & otherwise\end{cases}$$

When n = 0

$$\lambda\int_{0}^{\infty} xe^{-\lambda x}dx$$

$$\lambda\left[\frac{xe^{-\lambda x}}{-\lambda}-\int\frac{xe^{-\lambda x}}{-\lambda}dx\right]$$

$$\left[xe^{-\lambda x}+\frac{xe^{-\lambda x}}{-\lambda}\right]^{\infty}$$

$$\left[\frac{e^{-\lambda x}}{\lambda}\right]\infty=\frac{1}{x}\left[e^{0}-e^{0}\right]=\frac{1}{\lambda}$$

Mean E(x) = $\frac{1}{\lambda}$

Variance $v(x)=E(x^{2})-\left[E(x)\right]^{2}$

$$v(x)=\int_{0}^{\infty}x^{2}\lambda e^{-\lambda x}dx-\left(\frac{1}{\lambda}\right)^{2}$$

$$\left[\frac{x^{2}e^{-\lambda x}}{-\lambda}+2\int\frac{x^{-\lambda x}.x.dx}{\lambda}\right]_{0}^{\infty}-\frac{1}{\lambda^{2}}$$

$$=0+2\left[\frac{1}{\lambda^{2}}\right]-\frac{1}{\lambda^{2}}=\frac{1}{\lambda^{2}}$$

$$v(x)=\frac{1}{\lambda^{2}}$$

Standard deviation $\sqrt{v(x)}=\frac{1}{\lambda}$

For n=0, mean and standard deviation are equal.

For n = 1

Mean $E(x) = \int_0^\infty xe^{-\lambda x}\lambda x.\lambda dx$

$E(x) = \lambda^2 \int_0^\infty x^2 e^{-\lambda x} dx$

= $\lambda^2 \left[\frac{x^2 e^{-\lambda x}}{-\lambda} + 2\int \frac{e^{-\lambda x}}{-\lambda}.xdx \right]_0^\infty$

= $2\lambda \left[\int_0^\infty xe^{-\lambda x} dx \right]$

= $2\lambda \left[\frac{xe^{-\lambda x}}{\lambda} + \int \frac{xe^{-\lambda x}}{\lambda} dx \right]_0^\infty$

Mean E(x) = $\frac{2}{\lambda}$

Variance $V(x) = \int_0^\infty x^2 e^{-\lambda x}\lambda x.\lambda dx - \left(\frac{2}{\lambda}\right)^2$

$= \lambda^0 \int_0^\infty x^3 e^{-\lambda x} dx - \left(\frac{2}{\lambda}\right)^2$

$= \lambda^2 \left[\frac{x^3 e^{-\lambda x}}{-\lambda} + 3\int_0^\infty \frac{e^{-\lambda x}}{\lambda}.x^2 dx \right]_0^\infty$

$= 3\lambda \left[\int_0^\infty e^{-\lambda x} x^2 dx \right] - \left(\frac{2}{\lambda}\right)$

$= 3\lambda \left[e^{-\lambda x} x^2 dx \right] - \left(\frac{2}{\lambda}\right)^2$

$= 3\lambda \left[\frac{e^{-\lambda x} x^2}{-\lambda} + 2\int \frac{e^{-\lambda x}}{+\lambda} xdx \right] - \frac{4}{\lambda^2}$

$= 6\int_0^\infty e^{-\lambda x} dx - \frac{4}{\lambda^2}$

$= 6\left[\frac{e^{-\lambda x}}{-\lambda} + \int \frac{e^{-\lambda x}}{+\lambda} dx \right] - \frac{4}{\lambda^2}$

$$=6\left(\int_0^\infty e^{-\lambda x}dx\right)-\frac{4}{\lambda^2}$$

$$=\frac{6}{\lambda}\left(\frac{e^{-\lambda x}}{\lambda}\right)_0^\infty-\frac{4}{\lambda^2}$$

$$=\frac{6}{\lambda^2}\left(e^{-\lambda x}-e^0\right)-\frac{4}{\lambda^2}$$

$$=\frac{6}{\lambda^2}\times-1-\frac{4}{\lambda^2}$$

$$=\frac{6}{\lambda^2}-\frac{4}{\lambda^2}$$

$$V(x)=\frac{2}{\lambda^2}$$

V(x) = Variance = $\frac{2}{\lambda^2}$

Standard deviation = $\sqrt{V(x)}=\sqrt{\frac{2}{\lambda^2}}$

Standard deviation = $\frac{\sqrt{2}}{\lambda}$

When n = 2

Mean = $E(x)=\int_0^\infty xf\left(x\right)dx$

= $\int_0^\infty x\frac{e^{-\lambda x}\left(\lambda x\right)^2}{2!}\lambda dx$

$$=\frac{\lambda^3}{2}\left[\frac{e^{-\lambda x}x^3}{-\lambda}+3\int\frac{e^{-\lambda x}}{\lambda}x^2dx\right]$$

$$=\frac{\lambda^2}{2}\left[\int_0^\infty e^{-\lambda x}x^2dx\right]$$

$$=\frac{\lambda^2\times 2}{2}\left[\frac{x^2e^{-\lambda x}}{-\lambda}+2\int\frac{e^{-\lambda x}}{\lambda}x\ dx\right]_0^\infty$$

$$=3\lambda\left[\int_0^\infty e^{-\lambda x}dx\right]$$

$$=3\lambda\left[\frac{xe^{-\lambda x}}{-\lambda}+\int_0^\infty\frac{e^{-\lambda x}}{\lambda}dx\right]$$

$$=3\int_0^\infty e^{-\lambda x}dx=\left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^\infty$$

$$=\frac{3}{\lambda}$$

Mean = $E(x)=\int_0^\infty xf\left(x\right)dx$ = $\frac{3}{\lambda}$

V(x) = Variance $v[x]=\int_0^\infty\frac{x^2e^{-\lambda x}\left(\lambda x\right)^2}{2!}dx-\left(\frac{3}{\lambda}\right)^2$

$$=\frac{\lambda^3}{2}\left[\int_0^\infty x^4e^{-\lambda}xdx\right]-\frac{3}{\lambda^2}$$

$$=\frac{\lambda^3}{2}\left[\frac{x^4e^{-\lambda x}}{-\lambda}+4\int_0^\infty\frac{x^3e^{-\lambda x}}{\lambda}\right]_0^\infty-\frac{9}{\lambda^2}$$

$$=2\lambda^2\int_0^\infty x^3e^{-\lambda x}dx-\frac{9}{\lambda^2}$$

$$=2\lambda^2\left[\frac{e^{-\lambda x}}{-\lambda}x^3+3\int_0^\infty\frac{e^{-\lambda x}x^2}{+\lambda}dx\right]_0^\infty-\frac{9}{\lambda^2}$$

$$=6\lambda^2\left[\int_0^\infty e^{-\lambda x}x^2dx\right]-\frac{9}{\lambda^2}$$

$$=6\lambda\left[\frac{e^{-\lambda x}}{-\lambda}x^2+2\int_0^\infty\frac{e^{-\lambda x}}{\lambda}x\,dx\right]_0^\infty-\frac{9}{\lambda^2}$$

$$=12\left[\frac{xe^{-\lambda x}}{-\lambda}+\int_0^\infty \frac{e^{-\lambda x}}{\lambda}dx\right]_0^\infty-\frac{9}{\lambda^2}$$

$$=\frac{12}{\lambda}\int_0^\infty e^{-\lambda x}-\frac{9}{\lambda^2}$$

$$=\frac{12}{\lambda^2}\int_0^\infty\left[e^{-\lambda x}\right]-\frac{9}{\lambda^2}$$

$$=\frac{12}{\lambda^2}[0,1]-\frac{9}{\lambda^2}=\frac{12}{\lambda^2}-\frac{9}{\lambda^2}=\frac{3}{\lambda^2}$$

When n = 3

Mean = $E(x)=\int_0^\infty f(x)f(x)dx$

$$=\int_0^\infty \frac{xe^{-\lambda x}\left(\lambda x\right)^3\lambda}{3!}dx$$

$$=\frac{\lambda^4}{6}\int_0^\infty x^4e^{-\lambda x}dx$$

$$=\frac{\lambda^4}{6}\left[\int_0^\infty \frac{x^4e^{-\lambda x}}{-\lambda}dx\right]$$

$$=\frac{\lambda^4}{6}\left[\frac{x^4e^{-\lambda x}}{-\lambda}+4\int\frac{e-\lambda x}{\lambda}x^3dx\right]$$

$$=2\lambda^3\left[\int_0^\infty \frac{e-\lambda x}{\lambda}dx\right]$$

$$=\frac{2\lambda^3}{3}\left[\int_0^\infty \frac{x^3e\lambda x}{-\lambda}+3\int\frac{e^{-\lambda x}x^2}{\lambda}\right]_0^\infty$$

$$=2\lambda^2\left[\int_0^\infty e^{-\lambda x}x^2dx\right]$$

$$=2\lambda^2\left[\frac{x^2e^{-\lambda x}}{-\lambda}+2\int\frac{e^{-\lambda x}x}{\lambda}dx\right]$$

$$= \frac{2\lambda^3}{3}\left[\int_0^\infty e^{3-\lambda x} dx\right]$$

$$= \frac{2\lambda^3}{3}\left[\frac{x^3 e^{-\lambda x}}{-\lambda} + 3\int \frac{e^{3-\lambda x} x^2}{\lambda}\right]_0^\infty$$

$$= 2\lambda^2\left[\int_0^\infty e^{-\lambda x} x^2 dx\right]$$

$$= 2\lambda^2\left[\frac{x^2 e - \lambda x}{-\lambda} + 2\int \frac{e^{-\lambda x} x \; dx}{\lambda}\right]_0^\infty$$

$$= 4\lambda\left[\int_0^\infty e^{-\lambda x} dx\right]$$

$$= 4\lambda\left[\frac{e^{-\lambda x} dx}{-\lambda} + \int \frac{e^{-\lambda x}}{\lambda} dx\right]_0^\infty$$

$$= 4\left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^\infty$$

$$= \frac{4}{-\lambda}[0-1] = \frac{4}{\lambda}$$

Mean = $E(x) = \frac{4}{\lambda}$

$$V(x) = \int_0^\infty \frac{x^2 e^{-\lambda x} (\lambda x)^3 \lambda}{6} dx - \left(\frac{4}{\lambda}\right)^2$$

$$\frac{x^4}{6}\int_0^\infty x^5 e^{-\lambda x} dx - \frac{16}{\lambda^2}$$

$$\frac{\lambda^4}{6}\left[\frac{x^5 e^{-\lambda x}}{x} + \frac{5}{\lambda}\int e^{-\lambda x} x^4 dx\right]_0^\infty - \frac{16}{\lambda^2}$$

$$\frac{5x^3}{6}\left[\frac{e^{-\lambda x} x^4}{-\lambda} + 4\int \frac{e^{-\lambda x}}{\lambda} x^3 dx\right]_0^\infty - \frac{16}{\lambda^2}$$

$$\frac{5x^2 \times 4^2}{6}\left[\int_0^\infty \frac{e^{-\lambda x}x^3}{} dx\right]-\frac{16}{\lambda^2}$$

$$\frac{10\lambda^2}{3}\left[\frac{e^{-\lambda x}x^3}{-\lambda}+3\int\frac{e^{-\lambda x}x^2}{\lambda}dx\right]\frac{10}{\lambda}$$

$$10\lambda\left[\int_0^\infty \frac{e^{-\lambda x}x^2 dx}{\lambda}\right]-\frac{16}{\lambda^2}$$

$$\frac{20}{\lambda^2}-\frac{16}{\lambda^2}=\frac{4}{\lambda^2}$$

$$E(x) \ = \int_0^\infty xf(x)dx$$

$$\int_0^\infty \frac{xe^{-\lambda x}(\lambda x)}{24}dx$$

$$\frac{\lambda^5}{24}\int_0^\infty x^5 e^{-\lambda x}(dx)=\frac{5}{\lambda}$$

Variance $\int_0^\infty \frac{x^2 e^{-\lambda x}(\lambda x)^4 \lambda dx}{24}\int_0^\infty \frac{x^6 \lambda 5 e^{-\lambda x}}{24}dx$

$$\frac{30}{\lambda}-\left(\frac{5}{\lambda}\right)^2=\frac{30}{\lambda^2}-\frac{25}{\lambda^2}=\frac{5}{12}$$

In general we can write Mean of the above distribution is $\frac{n+1}{\lambda}$

Variance of above distribution is $\frac{n+1}{\lambda^2}$

Standard deviation of above distribution is $\frac{\sqrt{n+1}}{\lambda}$

CONCLUSION

In this paper proposed the Mean, Variance and Standard Deviation.

Mean of the above distribution is $\frac{n+1}{\lambda}$

Variance of above distribution is $\frac{n+1}{\lambda^2}$

Standard deviation of above distribution is $\frac{\sqrt{n+1}}{\lambda}$

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