Table 1: Types of changes in the cognitive structure

Chai (2008)	Piaget (1978) as in Pritchard and Woollard, (2011)
Adding new knowledge	Assimilation
Gap filling	
Conceptual change	Accommodation

A conceptual change strategy is based on the constructivist perspective of learning that learners have active role in building and re-structuring there cognitive structure (Sarar & Al-Migdady, 2014). Hence, error and alternative conceptions are expected as part of the construction process. Conceptual change is then defined as learning that changes an existing conception, including belief, idea, or way of thinking that belief to be erroneous or alternative conception (Davis, 2001).

Lee and Kwon (2001, p. 5) defined cognitive conflict as “perceptual state where one notices the discrepancy between one's cognitive structure and environment (external information), or between the components of one's cognitive structure (i.e., one's conceptions, beliefs, sub-structures and so on) which are in cognitive structure)”. Cognitive conflict occurs when an individual unable to apply his/her existing concept to solve a problem, and is thus confronted with a situation that motivates the learning of new concepts (Lee & Kwon, 2001). This being in a state of mental disequilibrium, can be detected by learners response to test items or class activity provided the teacher is aware and has the ability to do so. If learners derived to explore the problem systematically and carefully in a concept change approach so that they reconcile and settle their disturbed state successfully and hence results learning otherwise, can be a causes of dissatisfaction in the learning.

Conceptual change strategies found to treat cognitive conflict (usually, “alternative conceptions” or “misconceptions”) in basic science and of use widely since the early 1980s (Lee & Kwon, 2001; Posner, Strike, Hewson, & Gertzog, 1982). Nowadays, it has not only got attention by researchers in mathematics education (Assagaf, 2013) but also considered as helpful to overcome misconceptions and learning difficult in all subject areas (Davis, 2001).

One of the difficulties students encounter in the learning of calculus is overgeneralization and development of an alternative conception. Alternative conception can occur in any one status of prior knowledge: absence, incompleteness, or interference to the “to be learned” concept (Chi, 2008). Basically, alternative conception is not wrong thinking but it is a concept in embryo or a local generalization that learners make (Swan, 2001) or concept image that is not completely accurate to the scientific thinking (Keeley, 2012). For instance, thinking that limit value is the same as function value is a generalization developed from working on continuous functions at the introductory part and confusing continuity with connectedness due to the concept image about continuity from pre-calculus definition i.e. the pencil metaphor (Wangle, 2013).

Due to an alternative conception, an individual's concept images about a certain concept may not match to formal concept definition taught. If they do not match (as a result of their concept formation process) the concept definition taught, the individual face an obstacle in solving problems involving the given concept or hinder her/his further understanding. As learners encounter mismatch of their current knowledge (concept image) and what teachers or books says (formal concept definition), they develop an obstacle, which make them in a cognitive conflict. Thus, these inaccurate or partially accurate conceptions need to be resolved (Assagaf, 2013) in order to attain "equilibration" (equilibration as in Piaget's theory of cognitive development).

The concept change approach, at its early stage was criticized on the outlook of learners and knowledge. Some of the critics were: preconceptions can be resistant to change, learning specially overcoming difficulties is not always smooth, ignores non cognitive domains of learning, focuses on an approach that emphasizes and assumes logical and rational thinking (Pintrich, Marx & Boyle, 1993). Latter influenced by activist of social constructivism, conceptual change is no longer viewed as being focused on cognitive factors. Affective, social, and contextual factors also considered to contribute to conceptual change (Hewson, Beeth & Thorley, 1998). Some theorists (e.g. Duit, 1999) also suggest integrating concept change approach with cooperative learning strategy.

According to Chi (2008), even though the definition of concept change is somewhat seems clear, concept change as a learning strategy is not smooth. Chi (2008, p.61) further mentioned the following as a key issue to be considered for effective implementation of concept change strategy: in what ways is knowledge misconceived? Why is such misconceived knowledge often resistant to change? What constitutes a change in prior knowledge? How should instruction be designed to promote conceptual change?

According to Limon (2001), to attain concept change through cognitive conflict strategy attention should be given for the following key activities: make students aware of their existing concepts before instructional intervention, confront them with contradictory information, using anomalous data or discrepant events to replace prior concepts with scientifically accepted ones, and measure the resulting conceptual change.

Vosniadou and Verschaffel (2004, p.449) describe the following advantage of conceptual change approach in mathematics instruction:

> It can be used as a guide to identify concepts in mathematics that are going to cause students great difficulty, to predict and explain students' systematic errors and misconceptions, to provide student-centered explanations of counter-intuitive math concepts, to alert students against the use of additive mechanisms in these cases, to find the appropriate bridging analogies, etc.

This study is aimed to make assessment of the possible effect of the concept change approach with attention to these strength and treat of the strategy mentioned in the literature.

3. RESEARCH DESIGN

This study was conducted in sequential approach. First in a survey design the students' pre conception was assessed and analyzed qualitatively. Based on the conclusion drawn an intervention was prepared. On the second design, a single group was studied with observation (pre-test), treatment (concept change approach strategy) and observation (post-test) sequence. After difficulties are identified, it is believed that to teach students according to the concept change strategy including group practice, reflection and communication, assist them analyze errors and use it as spring board for further progression.

3.1 Sampling design

The focus of study was to assist students overcome difficulties and to enhancing their conceptual understanding in calculus. In doing so, natural science stream students in one University were taken as study population. Purposive sampling was employed in selecting students as respondent of the study. The students who were selected were enrolled in first year chemistry department for the academic year 2018-2019. They were 49 in number (F=16, M=33)

3.2 Instrument

The study employed two different types of data collection instruments. The first is the calculus concept test (pre-test) used to asses students error and misconception. The second is the intervention developed based on the identified pre-conception and the proposed conceptual change strategy. The test was used again as a post-test to assess the possible effect of the proposed and implemented strategy.

2.3 Data analysis

Both qualitative and quantitative methods of data analysis were implemented. At the beginning, the data collected from the pre-test were analyzed qualitatively (thematic analysis) to identify errors and misconceptions. The identified errors and misconceptions were used as an input to the development of the proposed strategy. The post-test result analyzed using t-test for paired group design with the help of SPSS version 25.

4. RESULT AND DISCUSSION

4.1 Error analysis of incoming students

The aim of the test was to determine how students conceive concepts in calculus. The section composed of five closed ended items and two open ended/work out items. In the closed ended items, the choice of each distracter has an implication on students' concept image, error type and level of knowledge. Each of these concept images that students possess were synthesized in more detail. Table 2 is summary of response for these five closed ended items and table 3 is summary of response for the two open ended items.

Table 2: Breakdown of students' choices to the five closed ended items

Item	N=238											
	A		B		C		D		E		NR	
	N	%	N	%	N	%	N	%	N	%	N	%
1	3	6	14	28	7	14	10	20	14*	28	2	4
2	5	10	6	12	4	8	14*	28	20	40	1	2
3	2	4	15	30	8	16	10	20	14*	28	1	2
4	10	20	23	46	2	4	12*	24	1	2	2	4
5	6	12	15*	30	13	26	6	12	7	14	3	6

* correct answer of the item

Table 3: Breakdown of students' choices to open ended items

Item	N=50							
	Correct		PC		Incorrect		NR	
	N	%	N	%	N	%	N	%
6a	12	24	13	26	13	26	12	24
6b	20	40	12	24	12	24	6	12
6c	22	44	0	0	17	34	11	22
6d	20	40	2	4	15	30	13	36
7a	15	30	0	0	22	44	13	26
7b	7	14	0	0	28	56	15	30

Based on the analysis made on the data gathered through the test in the specified area, the observed difficulties and error type of students in the study are summarized as follows:

- consider infinity as a number (actual value image) and evaluate $\infty * 0 = 0$, $\frac{0}{0} = 0$, and $\frac{0}{0} = \infty$,
- Influenced by arithmetic approach for items demanding an algebraic approach, evaluate rational functions before looking for any possible simplification to compute limit
- confuse the indeterminate form $\frac{0}{0}$ and undefined ($^{c}/_{0}$, for $c \neq 0 \in \Re$),
- think that limit value is necessarily a boundary, is not attainable, and limit is an approximation
- confuse existence of limit and being defined, in particular think that limit at a point is the same as the function value at the limit point and existence of limit is sufficent for being defined,
- provide correct answer for wrong reason,
- limit does not exist necessarily imply the function is unbounded,
- think that existence of limit is sufficient for continuity of a function at a point,
- hard to interpret result obtained from computation, and
- make procedural errors.

4.2 Discussion

- Most students think that limit at a point is just the function value at the limit point. This level of understanding is known as "action view of limit" (Carlson et al, 2010) and this action view of function than process based view is the main challenge to progress in calculus (Maharaj, 2012).

- Most students have an actual value image of infinity than potential. Jones (2015), states that actual infinity is valuable for infinite limit but not sufficient as “potential infinity” level of conception has much valuable to limit at infinity (p.108).
- Different types of algebraic manipulation errors, which rooted from pre-calculus algebra, were observed. According to Siyepu (2015, p.15) these difficulties root from prior learning practices that focuses on procedures and routine exercises than conceptual aspects.
- Good number of participants demonstrated misinterpretation of the indeterminate form. These misinterpretations together with action views of limit are main sources of difficulties in particular to limit of rational functions. Because as students test script revealed, after substitution when they get in indeterminate form $\frac{0}{0}$, good number of them conclude that either the limit is zero or the limit does not exist.
- Good number of students has no coherence and consistency in their work and have conflicting concept image about a concept. They have a limited concept image of limit of function, as a result their concept of limit fail into either all about an infinite process and nothing to do with finite value or limit is all about a finite value and nothing to do with infinite process.
- Most students over generalized that limit at a point is a substitution, every point of discontinuity is an asymptote. Most students’ knowledge is limited and seems fair only for continuous functions. Most students can compute limit or differentiate a function but they face a challenge to attach a meaning to the calculated value. Some students also lack to demonstrate correct symbolic manipulation and computations.

Overall, the data obtained revealed that most students lack the required pre university calculus knowledge. In addition, most of the observed errors occur due to the approaches used to introduce the concepts, the nature of activities and due to the dual nature of some concepts like limit and infinity.

5. INTERVENTION BASED ON THE ERROR OBSERVED

From a constructivist perspective of learning, error and misconceptions are opportunities for progression. Thus, based on these observed difficulties an intervention was prepared. The intervention was set of activities that aimed to overcome these observed difficulties through working on the activities. The activities were designed in a concept change approach so that students form cognitive conflict and in coming to come out of these conflict they overcome observed errors and knowledge gaps. Students worked individually and in group through these exercises. See appendix for some of the activities.

6. POSSIBLE EFFECT OF THE INTERVENTION

6.1 Comparisons of means

To analyze the possible effect of the intervention, a t-test statistics with the help of SPSS version 25 was used. To do so, first, data were explored in terms of normality and outliers by using the statistics software. Any divergence from normality was examined in terms of the standardized scores, skewness, kurtosis, and P-P plots (Tabachnick & Fidell, 2007). No outliers or missing scores were detected.

Next , to answer the research questions: paired sample *t*-tests were used to determine whether there were statistically significant between the pre-test and the post-test score within group internally. Table 4 presents paired sample statistics and Table 5 presents paired sample t-test.

Table 4: Paired Samples Statistics

Paired Samples Statistics					
		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	pre-test	21.3469	49	10.40543	1.48649
	post test	27.9388	49	10.14152	1.44879

Table 5: Paired Samples Test

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
pre-test - post test	-6.59184	5.46549	.78078	-8.16171	-5.02196	-8.443	48	.000

Accordingly, the data revealed that there is a statically significance difference between the pair of results for $p < 0.05$.

6.2 Conclusion

The main purpose of the study was to enhance students' conceptual understanding of calculus concepts through overcoming most frequently observed difficulties. Accordingly, exploring of difficulties at entry level was conducted. Based on those observed difficulties an intervention was prepared and administered. The result revealed a significance difference. Besides, students' group participation was increased. The study evidenced that by using students' current level of knowledge and error, it is possible to make them make sense and strive for progression. Error and misconceptions are good starting pointes to do so.

6.3 Recommendations

- Instead of blaming incoming students for their lack of the required pre understanding, it is recommended to use error analysis of their current understanding as spring board. Besides teachers should have to work on classroom exercises so that students get exposure to items that demand process level of conception, interpret results obtained from computation and focused on embedded concepts than rote learning.
- Further studies should have to be conducted to see the effect of concept change strategy in long learn.

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Appendix: Some of the intervention activities

Activity 1:

Two expressions concerning limits are given below:

a) $\lim_{x\to 0}\frac{\sqrt{x^2+9}-3}{x^2}$ and b) $\lim_{x\to \infty}\frac{\sqrt{x^2+9}-3}{x^2}$. Answer the question that follows a and b

i. Is it the same to find the limit of the given function as $x \to 0$ and $x \to \infty$? Explain your answer

ii. In finding the limit in question (a) the number 0 is substituted for x in the functional part and the result obtained becomes $\frac{0}{0}$. What conclusion can you draw from this result?

- The limit does not exist
- The limit is 0
- The limit is 1
- It is an indeterminate form
- The limit is ∞
- Any other, specify

iii. For question (b) write down any five numbers which you would substitute for x and explain why you think you have made an appropriate choice of such numbers?

iv. Calculate the limits as in a and b.

Activity 2:

How can we see if a function $y = f(x)$ has a limit L as x is approaching 0 ?

It is by:

- Calculating y for $x = 0$, i.e. calculate $f(0)$
- Calculating $f(1)$, $f(2)$, $f(3)$ and so on and observe the results
- Calculating $f(x)$ for $x = 1/2, 1/4, 1/8$ and so on
- Substituting x by 0 in the function formula, and calculate the value.
- Substituting numbers that are very close to 0 for x in the formula and look for the value of y.
- Substituting numbers that are very close to *0* for x in the formula and look for the value of y that is being approached as x values approach 0.

Activity 3:

3.1 Justify that $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e = 2.71828 \ldots$ Well, if you try to use direct substitution, what will happen?

3.2 Consider the function $f(x) = \frac{tanx}{x}$. How can you find the limit of f at $x = \frac{\pi}{2}$? Well, if you try to use direct substitution, what will happen?

3.3 Notice that in finding limit the three most common methods are substitution, rationalization and conjugate. Now, if any of these methods not work what will be your conclusion? Could it be necessarily limit does not exist?

Activity 4:

4.1 When you use words like "approaching" and "tends to", what do you mean? Do you think they seem to imply motion or you think of something moving? Justify.

4.2 Given a function f and a number c. Describe in your own words what it means to say that the limit of a function f as $x \to c$ is some number L?

4.3 Describe cases where limit of functions at a point fails to exist? Discuss all the cases exhaustively.

4.4 Explain the procedure to find the limit of $\lim_{x\to a} f(x)$, where $f(x)$ is a split-function given in symbolic form.

Activity 5: Consider the function $f(x) = \frac{x^3-1}{x-1}$

a. What is domain of f?

b. What is limit of f at $x = 3$?

c. The only place where $\frac{x^3-1}{x-1}$ and $x^2 + x + 1$ differ is $x = 1$. Why is it acceptable to interchange these two functions even though we are trying to find limit at $x = 1$?

Activity 6: A function f behaves in the following way near $x = 3$: As x approaches 3 from the left, $f(x)$ approaches 2. As x approaches 3 from the right, $f(x)$ approaches 1.

For the above situation you are required to:

a. Draw a sketch to illustrate the behavior of f near $x = 3$.

b. Write the 2nd and 3rd sentences in symbolic form.

c. Check that your symbolic form agrees with the sketch you drew.

d. Determine with reasons if $\lim_{x\to 3} f(x)$ exists.

Activity 7: Let $f(x) = \begin{cases} a\frac{sinx}{x-|x|}, & if\ x < 0 \\ e^{-x} + cosx, & if\ x \geq 0 \end{cases}$ if f is continuous at $x = 0$, then what is the value of a? The following steps are part of the procedure to answer this problem. Give reason why each of these steps is logical

Step	Reason
$\lim_{x\to 0^-} \frac{asinx}{x-\lvert x\rvert} = \lim_{x\to 0^+}(e^{-x} + cosx)$	
$\lim_{x\to 0^-} \frac{asinx}{x-\lvert x\rvert} = \lim_{x\to 0^-} \frac{asinx}{2x}$	
$\lim_{x\to 0^-} \frac{asinx}{2x} = \frac{a}{2}$	
$\lim_{x\to 0^+}(e^{-x} + cosx) = 2$	
Hence, $a = 4$	