The role of transshipment in a single-echelon and multi-retailer inventory system in a wholesale company in Tunisia

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ABSTRACT

In this paper, we deal with the case of a network made up of a distribution center that supplies several retailers.
We assume that the demand D_i ($i = 1, 2, 3$) at site i follows a normal distribution with mean μ i and standard deviation σ_i (known). Retailers work together in the event of a shortage of inventory by shifting the necessary amount of transshipment to meet expected customer demand.
The model is an extension of previous work by (Meissner and Rusyaeva (2016)) where transshipment between more than two retailers is permitted. Such an extension introduces an additional complicating element which is the strategy of lateral transfer of product in the following two situations: (1) when at least one retailer faces a shortage of stock at the end of a periodicity noted as "T When two or more other retailers have excess stock, (2) when two or more retailers are faced with insufficient stock and they request the missing quantity from only one retailer who has excess.
The objective of this paper is to study the performance of a distribution system made up of a central warehouse and three retailers and to assess the collaboration both at the level of Average Global Profit and of the Average Global Desservice level.

Keywords: Transshipment, discrete event simulation (DES), Vendor-managed inventory, metamodel-based simulation, Desirability function approach, partial-pooling threshold.

1. Introduction

In the inventory system consisting of two-echelon of neighboring multi-retailers are located at shorter distances than the supplier, one can request stock from others when it is out of stock, that is to say that emergency lateral transshipment between these retailers is commonly practiced to improve the rate of fulfilled orders.
In this paper, we develop an inventory model composed of multi-retailer with continuous review by applying the stocking policy ($\boldsymbol{R}, \boldsymbol{S_i}$), for consumable products when emergency lateral transshipments between these retailers are allowed. Approximations are derived for the expected level of average overall profit and the average global Desservice rate of retailers. Numerical examples are presented to illustrate the effects of emergency lateral transshipment on the performance criteria of inventory systems. The results of the experiments carried out by the simulation using the ARENA 16.0 software and then optimized using its OptQuest tool indicate that the emergency lateral transshipment leads to a significant decrease in the average global Desservice rate of retailer orders, which leads to a overall gain improvement of this centralized system.

In this research, we study in Section 2 the literature review of research.We studied the problem description in Section 3 to Section 4. Numerical results and interpretation are presented in Sections 5. Conclusion will be presented in Section 6.

2. Literature review

2.1. Lateral Transhipmentpolicies

The literature on lateral transshipments can be divided in two main categories that differ in the timing of transshipments. The first category is known as proactive transshipment or preventive lateral transshipment, where lateral transshipments can be limited to take place at predetermined times before all demand is realized. In this case, lateral transshipments are used to redistribute stock amongst all stocking points in an echelon at programmed moments. Consequently, preventive lateral transshipment is appropriate when the transshipment costs are comparatively low compared to the costs associated with holding large amounts of stock and with failing to meet demands immediately. The second category which is called reactive transshipment, known also as emergency lateral transshipment, lateral transshipments can take place at any time to respond to stock outs or potential stock outs. In reactive transshipment, lateral transshipments are realized after the arrival of demand but before it are satisfied. If there is inventory at some of the stocking locations while some have backorder, lateral transshipments between stocking locations can work well. Some authors combine reactive transshipment and proactive transshipment policies together (known as service level adjustments) to reduce the risk of stock outs in advance and efficiently respond to actual stock outs. In fact, emergency lateral transshipment responds to actual stock outs while preventive lateral transshipment reduces the risk of possible future stock outs. Transshipment has been considered in the literature as a tool to balance inventory among locations in the same echelon to reduce shortage. Lateral transshipment policies can be classified into reactive (corrective) and proactive transhipment (preventive), Paterson et *al.,* (2011).

The first is that of emergency transshipment (transshipment reactive); it corresponds to the Transsipment carried out following an actual stock-out at a retailer resulting from the arrival of a demand. In the literature, several research studies are aimed at studying this approach. Most past studies considered reactive transshipment, in which transshipment occurs when an inventory shortage is, realized [Herer et *al.,* (2006), Yao et *al.,* (2016), Park et *al.,* (2016)]. In these studies, the transshipment time was considered negligible to make the problem tractable.

Paterson et *al.,* (2012) investigated the problem of a multi-level stock system composed of N- retailers, in the event of an actual retail outage due to a random customer demand arrival. They proposed a reactive approach to solve this problem.

Reyes et *al*., (2013) have studied the same problem as Paterson et *al.,* (2012) by focusing their research work on the impact of emergency transshipment on inventory management in this system in case of an actual stock-out, and they concluded that responsive transshipment can reduce costs and improve service r.ates by minimizing the amount of customer order lost.

Kim and Sarkar, (2017) proposed that time is one of the crucial elements of competition, customers get impatient and less tolerant of back orders. Partially unsatisfied orders are a common phenomenon in the retail trade. It has an obvious effect on the corrective transshipment performance because the latter is executed at the end of the sales season in the event of a stock shortage.

Dehghani and Abbasi (2018) considered an aged-based lateral shipment policy for the case of perishable items. They targeted the transshipment of blood units between hospitals. They developed partial differential equations to derive and solve a joint distribution problem that allowed them to determine the optimal inventory level at each location with transshipment based on the age of stock. They also showed that their approach could bring additional savings to a similarly structured distribution channel**.**
Yi et *al.,*(2020)studie optimal lateral transshipment and replenishment decisions under a decentralized setting. We construct a multi-stage stochastic model that captures demand uncertainty and customer switching behavior. We demonstrate that, similar to the centralized setting, the optimal transshipment decision follows a double-threshold structure.
The optimal replenishment quantities are determined under two pricing mechanismsindividual mechanism (IP) and negotiated mechanism (NP).

The second approach is that of proactive (or so-called proactive) transshipment, which is a redistribution of stocks at the beginning or end of each supply cycle but before customer demand is realized.

There is a vast literature that is interested in this type of transshipment approach. Preventive transshipment research is dominated by periodic review, because at the beginning and end of each period it is necessary to periodically check the quantities stored to attribute a redistribution of these quantities. In this regard, Agrawal et *al.,* (2004) envisioned a two-step inventory system in which they aimed to rebalance the quantity stored at a predetermined time before the demand was made and they presented a formulation dynamic programming to determine the best decisions. Van et *al.,* (2009) studied the problem of a two-tier stock system. They applied the Markov process to solve it by applying preventive transshipment on a specific date. Paterson et *al.,* (2010) analyzed a multi-warehouse inventory system that follows inventory policy (S-1, S) combined with the proactive transshipment policy. They assumed that the cost of transshipment is fixed, and they aimed to set the optimal time of the redistribution of stock to minimize the breakage, which entails a minimization of the global cost.

Some research projects have studied the inventory problem with preventive transshipment in a decentralized system. Li et *al.,* (2013) analyzed an inventory system with two storage depots which uses proactive transshipment as an approach to deal with the gap between demand and supply. A bidirectional an income sharing contract has been proposed to coordinate the transhipment quantities between the two entropots. Abouee-Mehrizi et *al.,*(2015) proposed a proactive transshipment model to minimize the mismatch between supply and request. They considered a multi-period inventory with a finite horizon system for two locations and optimal determination of joint replenishment and transhipment policies.

Dan et *al.,*(2016) developed a two-period order and pricing model with preventive transshipment and conditional return. To reduce the imbalance within the system, the manufacturer controlled preventive transshipment between two independent retailers.Feng et *al*., (2017) addressed the problem of the stock system by applying preventive transshipment. A heuristic combined with dynamic programming algorithms has been proposed to solve the problem. They proposed a non-linear model for a supply chain with transshipment between buyers who had limited warehouse capacity. They have resulted that transshipment increased the rate of use of storage capacity.Meissner and Senicheva, (2018) studied a multi-site, multi-period storage system with proactive (preventive) transshipment and approximate dynamic programming used determine an optimal order policy and transshipment policy.

To significantly improve a purely reactive transshipment policy, it would be possible to combine it with another proactive policy; this will be named by “Hybrid transshipment policy”.Glazebrook et *al.,*(2015) proposed a hybrid lateral transshipment policy such that the transshipment decisions are made when a location faces a shortage that resembles a reactive transshipment policy, however, the quantity of transshipment can exceed the current shortage to avoid future imbalance in the inventory system. They employed dynamic programming to solve their model, using a heuristic to approximate the future cost of a decision.Dijkstra et *al.* (2019) consider the case that the return products ordered online at any offline store may result in unbalanced inventories. To deal with these unbalanced inventories, they study the optimal transshipment policy and prove that it can reduce the cost.

2.2. *Transshipment Direction*

The literature on unidirectional transshipment for a supply chain is, however, scarce. Seifert et*al*.,(2006) studied unidirectional transshipment integrating direct and indirect sales channels through a traditional retail store and a decentralized virtual store. They analyze how the supply chain of a single manufacturer and several identical retail stores can be coordinated by taking into account a combination of wholesale prices, inventory subsidies and transfer payments. Dong et*al*., (2012) studied a multi-level framework considering a contract manufacturer and two inventory locations which differ in scale and scope such that transshipments are performed only unidirectional to analyse information asymmetry within the context of transshipments. He et*al*., (2014) studied a dual channel supply chain with unidirectional transshipment policies between retailer and manufacturer under endogenous and exogenous transshipment prices. The setting in both papers is somewhat different to our horizontal setting as they consider unidirectional transshipments between different echelons.

Toyasaki et*al*., (2017) considered bidirectional and unidirectional transshipment of relief items in a decentralised humanitarian supply chain under correlated demands. However, since they consider a supply chain network in the non-commercial setting, their model shows significant differences to the commercial setting in terms of cost and price parameters. But, in centralized systems, most publications focused on bidirectional transshipment in supply chains.

This work assumes that transshipment is mutually beneficial for all retailers and object to maximize the global profit of the system and no longer of such a retailer.

Rudi and *al.,* (2001) show that the decentralised system can be coordinated by appropriately set transshipment prices. However, Hu et*al.*, (2007) provide examples which show that such coordinating prices may not exist in several cases. Especially with increasing asymmetries in the economic parameters for the two locations, coordination of bidirectional transshipments may not be possible by varying the transshipment prices. Li et*al.*,(2013) discuss the coordination problem of preventive bidirectional lateral transshipments between two independent locations and propose a bidirectional revenue sharing contract to coordinate the system.

Park et *al.,* (2016) extend the transshipment models of Rudi et*al.*, (2001) and Hu et*al*., (2007) by considering uncertain capacity of the supplier. They find that the sufficient condition for the existence of coordinating transshipment prices is more restrictive under supply capacity uncertainty and limitation than in the case of infinite capacity.

Li and Li, (2017) discussed the impact of bargaining power in a two-tier supply chain consisting of a manufacturer and two symmetrical retailers with bidirectional transhipment between them.

2.3. *Nature of stock management policy*

Most research work focuses on policies (No Pooling) and (Complete Pooling). For the first policy of transshipment, we cite some research (Guan and Zhao, 2010, Glock, 2012) and for the second policy of transshipment, we give as example (Bouma et *al.,* 2014).

First, we are interested in the problem of transsipment cooperated with the stock management policy (R, S).
In this field, at the end of each basic period, the stock is evaluated, the possible emergency transshipments are then carried out simultaneously and a supply order is placed if it is a revision period. This work generally adopts the deferred claims hypothesis. Recall that the policy (R, S) is particularly appropriate under the assumption of negligible command / setup costs.
Examples of work emphasizing the importance of politics (R, S), Banerjee et *al*., (2003) and Burton and Banerjee, (2005), which focused on the evaluation, by the and 2, 4, and 8 retailer site configurations, the benefits of policy-based transshipment (Complete-Pooling), and those of preventive transshipment.
The research of Herer et *al*., (2004)) focuses on the study of a stock system composed by multi-retailers that are not identical in terms of costs, without constraints of carrying capacity to achieve a reactive transshipment. The random demands arriving at the warehouses are supposed to be correlated (the demands are independent, identically distributed (*i.i.d*)).
The work of Özdemir et *al.,* (2006)) focused on the research of (Herer et *al*., 2004) considering transport capacity constraints according to which the transshipment quantities between deposits located at the same level are limited by the capacity of the means of transport. These researchers have developed an effective stochastic approximative approach using Monte Carlo simulation. The numerical results show that transport capacity constraints increase the global cost as well as alter the distribution of inventory throughout the network.
The same problem studied by (Özdemir et *al.,* (2006)) was also treated by (Ekren and Sunderesh, (2008)) applying the simulation-optimization method of resolution. The optimization procedure is performed by the OptQuest of the ARENA ® software.
Hu et *al.*, (2007) studied a storage system consisting of two retailers and they focused on emphasizing the non-coordination of transshipment prices.
Archibald et *al*., (2009), for their part studied a model composed of multi-retailers not identical in terms of costs, without constraints of transport capacity to achieve a transshipment. The demands arriving at the sites follow the fish law (the demands are independent, identically distributed (*i.i.d*)). To solve this problem they use Markovian resolution methodology.
Pazhani et *al*., (2015) focused in their research work on reducing the global cost of the storage system by minimizing the cost of disruption (minimizing the service rate) and transportation costs, and reducing the cost of transportation. improving the efficiency of the supply chain by making the best decision by selecting the optimal supplier under a stochastic demand constraint.
Second, we focus on the relationship between transshipment with stock management policy (s, Q)
About this, for work that has adopted the continuous revision policy, the system (s, Q) is the most commonly used because it is relatively simple. Investigations have been conducted under the two assumptions of lost-demand systems and delayed-demand systems.
Evers (2001), developed two heuristics to determine the conditions in which transshipments generate benefits for the stock system.
The first heuristic seeks to solve the problem of the transshipment of a single unit and the second addresses the transshipment of multiple units (multiple sites). The all-or-nothing transshipment policy is adopted in the (Evers (2001)) model with a linear transshipment cost, depending solely on the quantity transferred.

The research (Minner et *al.*, (2006)) focuses on a relaxation of the hypotheses of (Evers, (2001)) by accepting transshipments by quantities lower than those demanded and by adding a fixed cost per satisfied query. They also completed the model by taking into account the cost of supply as well as any possible costs of disruption as a result of the transshipment decision.

Satyendra and Venkata, (2005) studied a storage system (s, Q) composed by two-retailers assuming that the demand is random and follows the Normal N law (for that they applied the method of resolution by Simulation for search for the best solution in terms of global cost and rate of service Olssen (2009, 2010) was interested in solving the problem of "unidirectional lateral transshipment" in (s, Q) or (S -1, S) with deferred or lost demands.

Olssen (2015) studied a storage system (s, Q) composed of a distribution center and two retailers, he applied the analytical resolution method to find the optimal solution by applying the policy of transshipment (Partial Pooling).

We focus- on the cooperation between the problem of transshipment with the stock management policy (S-1, S). In this context, the study by (Wong et *al*., (2005)), is one of the few to have assumed that the time of non-negligible transshipment and a delayed transshipment (ie in case of rupture at a warehouse, if no deposit has stock available so the transshipment is delayed (put on hold) until the stock becomes positive in one of the storage sites).

Liu and Lee (2007) focused their research on a single-level, multi-product and multi-retail stock system. They emphasized the influence of partial transshipment on reducing global cost by applying the Markovian method of resolution.

Paterson, et *al.,* (2012) analyzed an inventory system consisting of a single-level, single-product and two-retailers. They demonstrated the importance of making a decision to make the transshipment only if the stock position is above a set threshold. To solve this problem, he applies the analytical resolution method.

Seidscher and Minner, (2013) examined policy (S-1, S) in a stock system composed of a distribution center and N-Retailers, to determine an optimal trans-shipment policy, they applied, first Instead, the policy reacts to minimize the out-of-stock rate, but they deduce that the amount of unsatisfied order is high. For this, they have combined this policy of transshipment with another proactive, which results in an efficient improvement of the optimal result in terms of cost and rate of service.

Patriarca et *al.,* (2016) studied a two-tier stock system, the first includes a distribution center and a maintenance department for repairable parts. The second echelon contains a large number of retailers. First, they applied Complete-Pooling when using transshipment, then they set a threshold beyond which they would make the decision to apply such transshipment.

Finally, we aim to study the transshipment problem with stock management policies (s, S) and (R, s, S)

In this area, the study of transshipment for stock systems (s, S) or (R, s, S) has given rise to relatively less work, probably because of its more complex nature.

Hu et *al*., (2005) examined the policy (R, s, S) in a stock system composed of a distribution center and multiple-retailers with centralized stock management at the distribution center level to improve the overall performance of the system whole. The assumptions considered in their model are very restrictive: zero supply and transshipment times, identical demand parameters, identical costs and infinite time horizon. In this framework, the authors proposed a dynamic programming approach to find the approximate optimal policy (s, S) of the entire system at the distribution center level.

Tlili et *al*., (2010) examined the policy (R, s, S) in a two-step inventory system, the first contains a distribution center with infinite storage capacity and the second composed of multi-retailers. Their research aimed to reason the benefits of complete-pooling and those of partial-pooling on cost reduction. To solve this problem, they applied the "Simulation-Optimization" resolution method and they showed that "partial-pooling" is more efficient than "complete-pooling", because with a partial transshipment, there remains such a quantity in deposit in overstock position, which may reduce the amount of order lost; this will improve the optimal result in terms of global cost and service rate by reducing the unsatisfied amount of customer demand.

Previous works have tended to assume that the demand function is linearly dependent on variables such as retail price or promotion cost, and that the constant term of the function, which is usually referred to as the initial market share, is disrupted by a variation (Shen and Li, 2016). We argue that the conventional technique of modeling a demand disruption is not suitable for characterizing disruption of stochastic demands. The conventional characterization of demand disruption is to assume there is an additive variation on the experienced demand value Shen and Li, (2016). However, when a demand is stochastic, it is hard to recognize whether an additive difference between the materialized demand value and the experienced demand value is due to the demand disruption or the essential uncertainty of the stochastic demand. So it is necessary to develop an alternative method to characterize the disruption of stochastic demands. In addition, in the presence of today's economic globalization, consumer demands are becoming even more unstable since they can be disruptedvery frequently, and even continuously (Grossman, 2016; Wolcott, 2016). This fact requires that the desirable characterization of disruption of stochastic demand should not only give the disrupted value biasbetween materialized value and the experienced value, but also reflect the decreasing systemic stability.

Xiao and Shi (2016) examined the problem of dual channel SC coordination where the manufacturer's production process works to a random yield rate. Since in this situation shortages are common, optimal decisions and coordination in SC are significant.They proposed two priority strategies to optimize decision variables.

Ji et *al*., (2017) considered demand disruption in a two-stage supply chain from the manufacturer to the retailer and then to the consumer, with a transshipment-before-buyback contract. This contract was also investigated for a supply chain of two retailers and a manufacturer and showed that it was beneficial for all parties to enter this contract. Their results also showed that a predetermined or negotiated transshipment price could benefit all parties where there is a disruption in demand and that a buyback guarantee does not influence transshipment price despite a manufacturer's incentive.

3. Mathematical modeling

3.1. Mathematical model

3.1.1. Hypotheses

- ***n***: the number of retailers which set at three;
- ***i***: the index of retailers, with *i* = 1, 2, 3;
- The demand of each retailer *i* is uncertain, stationary and independent of the demands of the other warehouses (*i.i.d*),
- The stock control is done periodically according to the storage policy (R, S_i),
- At the start of each supply cycle, an order of size Q_i is placed to reach the replenishment level S_i,
- The distance between the retailer and the central warehouse is very long, which implies a long lead time and a high procurement cost,
- The distance between the different retailers is short, because they are located on the same level, so that the transshipmnet time will be negligible,
- The cost of transshipment changes from one pair of retailers to another, but it remains low,
- In the event that depot *1* faces an effective out of stock and if the warehouse of the same level *2* or *3* has a surplus of stock, then a "lateral transshipment" of the necessary quantity will take place from *2* to *1* and / or from *3* to *1* to respond in a more efficient manner and at the right time to the random customer demand of retailer 1: this is the case with “Transshipment-Réactif” which aims to eliminate a feasible disruption. Otherwise he can place an order of size Q_i to the central warehouse.

But, before applying transshipment, it is necessary to study the following steps:

- Control the inventory level of each storage depot;
- Observe the customer demand of each retailer;
- If there is an actual stock shortage in such a warehouse, the stock of the one belonging to the same level must be checked;
- If the retailer at the same level has excess stock and can fill the break from the other site, this will require the application of transshipment;
- When retailers collaborate with each other with transshipment, the necessary policy must be taken (“Complete-Pooling”, “Partial-Pooling”);
- The transshipment will be applied at a very low cost compared to that of an emergency order from the central warehouse;
- Satisfy aggregate demand;
- Any unfulfilled request after the application of the transshipment will be lost;
- Determine the stock position after the demand has been satisfied by transshipment.

3.2. The main simulation models for the different transhipment strategies

Having decided that sites will fully share their stocks in the event of a risk of shortage, the objective of this paragraph is to determine how that risk can be most effectively shared in the event that only one site has a shortage of stocks while the other two have a surplus, or two sites run out of stock and they request the necessary amount from the third site which has excess stock. As there can be two shippers or two receivers of a surplus of stocks that will be shared between them, it is necessary to define a lateral transfer strategy of product which will make it possible to determine the channels and quantities to be transferred in the event of 'a shortage in the stock. In this model, three strategies were proposed: the Random Transshipment Strategy, the strategy according to the distance between the retailers which are named by the Strategy according to the Proximity of the retailers (Retailer Proximity Strategy) and the Strategy according to the Risk level (Risk Balancing Strategy).

The Arena simulation software was used to develop the simulation model of the supply network. Arena, developed by Rockwell Automation, is a simulation and automation software based on SIMAN processor and simulation language.

- **Random Transshipment Strategy:**

- If Retailer *3* is faced with an actual out of stock, (PS_{3T}<0), while, warehouses *1* and *2* are in a surplus stock position (PS_{1T}> 0 and PS_{2T}> 0), the source site which extradites the quantity needed to decrease the number of lost orders at warehouse *3* is chosen at random.

- If warehouses *1* and *2* face an effective stock shortage (PS_{1T}<0 and PS_{2T}<0), while the storage site *3* has a surplus of stock (PS_{3T}> 0), the choice of the depot which receives the product is done arbitrarily and without constraint.

Taking for example, if retailer *1* faces a shortage of stock, the choice of donor of the quantity of the necessary transshipment will be carried out randomly, then, in case of "Complete-Pooling", if,

$$\mathbf{X_{21} \leq PS_{2T} \quad and \quad X_{31} \leq PS_{3T} \quad SO \qquad X_{21} = X_{31} = (D_{1T}\text{-}PS_{1T})} \qquad (1)$$

For the first transshipment strategy called “Random Transshipment Strategy” and more precisely for the “Complete-Pooling” transshipment policy, the modeling by the ARENA 16.0 software can be presented in figure 1.

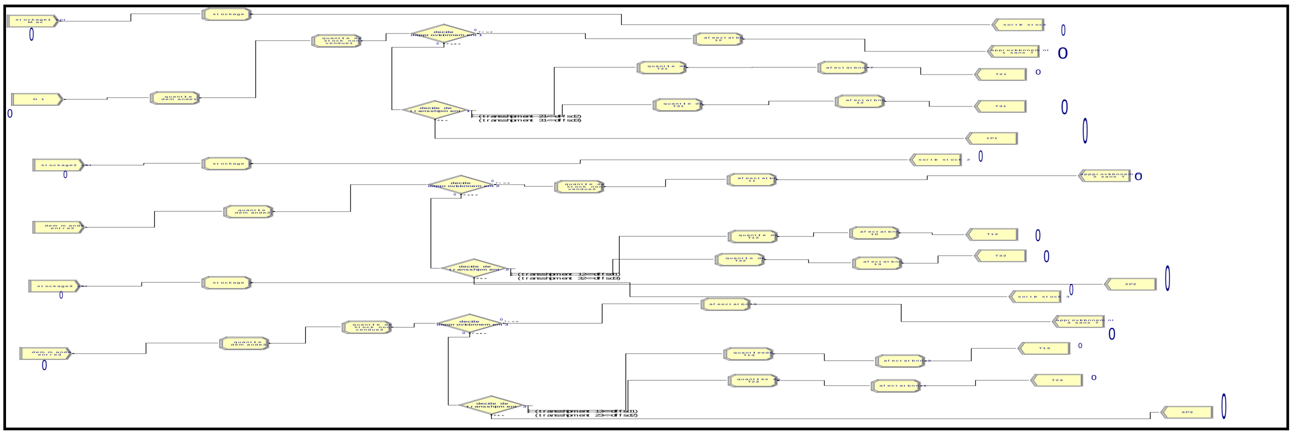

Figure 1 : the simulation model SC: Complete-Pooling for Random Transshipment Strategy

While, for the “Partial-Pooling” policy,

$$\begin{cases} \text{If } (PS_{2T} - \text{threshold}_{2T})>0 & \begin{cases} \text{If } (D_{1T} \text{ -}PS_{1T}) \le (PS_{2T} - \text{threshold}_{2T}) \text{ So } X_{21}=D_{1T} \text{ -}PS_{1T} \\ \text{Else} \quad X_{211}=(PS_{2T} - \text{threshold}_{2T}) \end{cases} \\ \text{And /Or} & \\ \text{If}(PS_{3T} - \text{threshold}_{3T})>0 & \begin{cases} \text{If } (D_{1T} \text{ -}PS_{1T}) \le (PS_{3T} - \text{threshold}_{3T}) \text{ So } X_{31}=D_{1T} \text{ -}PS_{1T} \\ \text{Else} \quad X_{311}=(PS_{3T} - \text{threshold}_{3T}) \end{cases} \\ \text{Else order lost} & \end{cases} \tag{2}$$

For the “partial-pooling” transhipment policy, the modeling by the ARENA 16.0 software can be presented in figure 2.

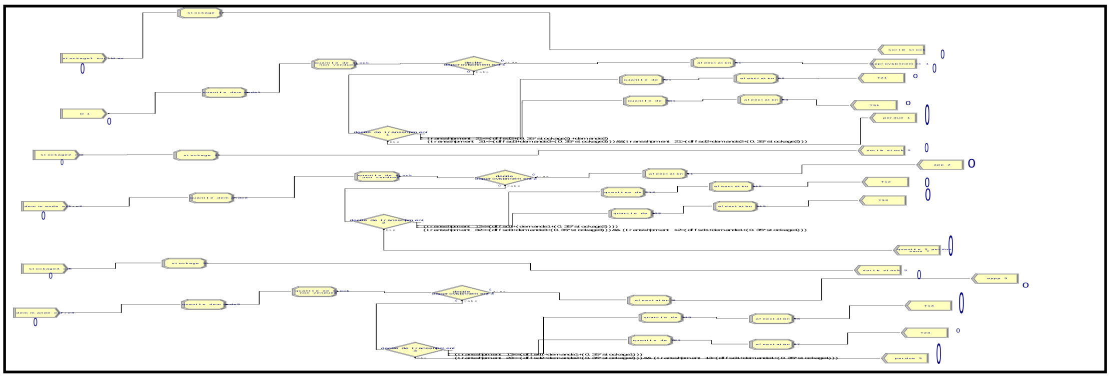

Figure 2: the simulation model SC: Partial-Pooling for Random Transshipment Strategy

- **Retailer Proximity Strategy**:

In our research work, we introduce the notion of the distance between the storage sites located at the same level; this gives priority to the nearest depot to transfer or to receive the necessary quantity of transshipment. In fact, we are adding another strategy called the “retailer proximity strategy” which allows to:

- If site 1 is confronted with an effective out of stock ($\boldsymbol{PS_{1T}}$<0) while 2 and 3 have a surplus of available stock ($\boldsymbol{PS_{2T}}$> 0 and $\boldsymbol{PS_{3T}}$> 0), the first shipper of the quantity necessary to eliminate the shortage in 1 is the closest one between the other two storage sites 2 and 3.

So for the first “Complete-Pooling” transshipment policy

- If $d_{21}<d_{31}$

And if $(X_{21} \leq PS_{2T})$ So $X_{21} = (D_{1T} - PS_{1T})$ **(3)**

And if $(X_{21} > PS_{2T})$ and $(X_{31} \leq PS_{3T})$ So $X_{31} = (D_{1T} - PS_{1T})$

- If $d_{31}<d_{21}$

And if $(X_{31} \leq PS_{3T})$ So $X_{31} = (D_{1T} - PS_{1T})$ **(4)**

And if $(X_{31} > PS_{3T})$ and $(X_{21} \leq PS_{2T})$ So $X_{21} = (D_{1T} - PS_{1T})$

But, for the second transhipment strategy called “**Retailer Proximity Strategy”** and specifically for the “Complete-Pooling” transshipment policy, the modeling by the ARENA 16.0 software can be presented in figure 3.

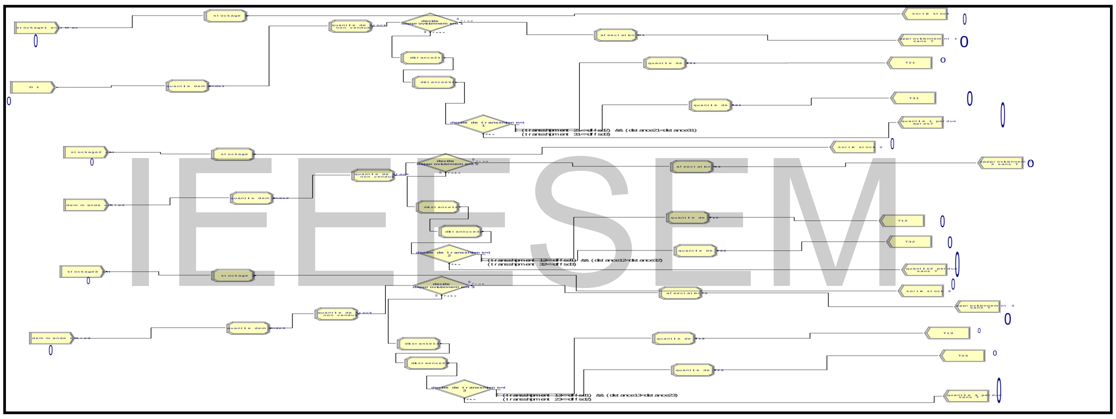

***Figure 3: the simulation model SC: Complete-Pooling for* Retailer Proximity Strategy**

But, for the second policy, "Partial-Pooling":

- If $d_{21}<d_{31}$

$$\begin{cases} \text{If } (PS_{2T} - \text{threshold}_{2T})>0 & \begin{cases} \text{If } (D_{1T}\text{ -}PS_{1T}) \leq (PS_{2T} - \text{threshold}_{2T}) \text{ So } X_{21}=D_{1T}\text{ -}PS_{1T} \\ \text{Else} \quad X_{211}=(PS_{2T} - \text{threshold}_{2T}) \end{cases} \\ \text{Else} & \\ \text{And If } (PS_{3T} - \text{threshold}_{3T})>0 & \begin{cases} \text{If } (D_{1T}\text{ -}PS_{1T}) \leq (PS_{3T} - \text{threshold}_{3T}) \text{ So } X_{31}=D_{1T}\text{ -}PS_{1T} \\ \text{Else} \quad X_{311}=(PS_{3T} - \text{threshold}_{3T}) \end{cases} \\ \text{Else order lost} & \end{cases} \quad \textbf{(5)}$$

- If $d_{31}<d_{21}$

$$\begin{cases} \text{If } (PS_{3T} - \text{threshold}_{3T})>0 & \begin{cases} \text{If } (D_{1T} - PS_{1T}) \le (PS_{3T} - \text{threshold}_{3T}) \text{So} X_{31}=D_{1T} - PS_{1T} \\ \text{Else} \quad X_{311}=(PS_{3T} - S\text{threshold}_{3T}) \end{cases} \\ \text{Else} & \\ \text{And if } (PS_{2T} - \text{threshold}_{2T})>0 & \begin{cases} \text{If } (D_{1T} - PS_{1T}) \le (PS_{2T} - \text{threshold}_{2T}) \text{So } X_{21}=D_{1T} - PS_{1T} \\ \text{Else} \quad X_{211}=(PS_{2T} - \text{threshold}_{2T}) \end{cases} \\ \text{Else order lost} & \end{cases} \tag{6}$$

Whereas, for the second transshiopment policy, "Partial-Pooling", the modeling modeling by the ARENA 16.0 software can be symbolized by the figure 4.

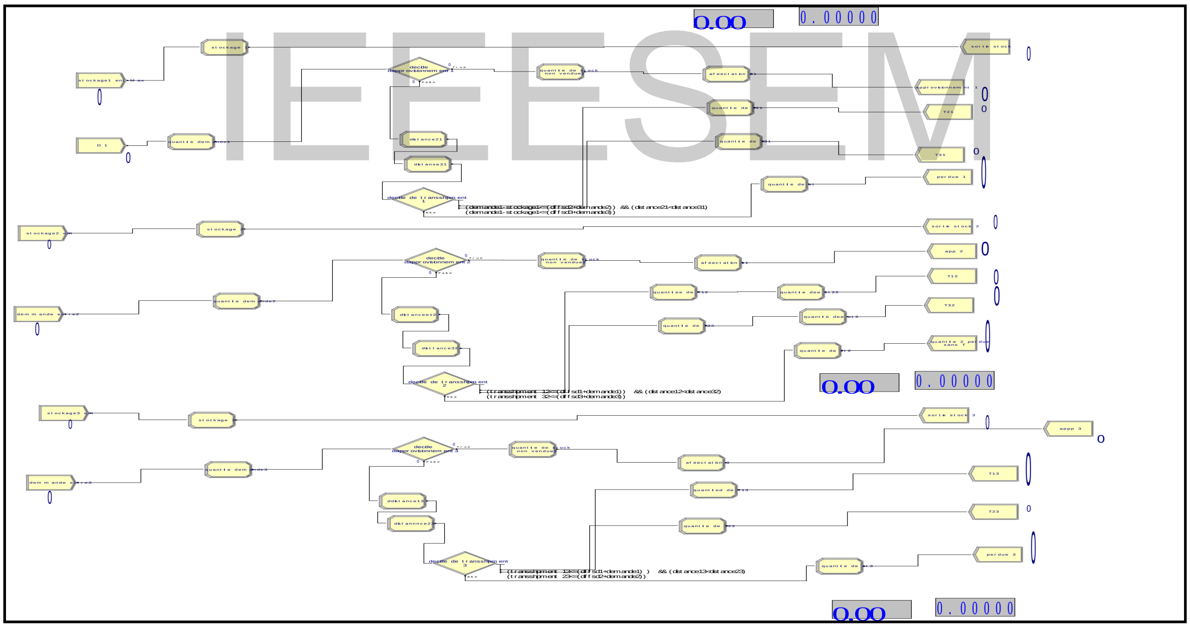

Figure 4: the simulation model SC: Partial-Pooling for **Retailer Proximity Strategy**

➢ **Risk Balancing Strategy:**

In fact, in the real world, to better cope with the shortage, you have to take into account the risk of the stock shortage. For this, we add another strategy called “the strategy of transshipment according to the confrontation of the risk”. It aims to redistribute the stock for all the same level retailers who collaborate with each other to better improve the Global Average Profit of the entire system while minimizing the Average Global Servicing Rate as much as possible.

This strategy requires that:

- If depot *1* faces a shortage of stock (PS_{1T}<0), whereas, warehouses *2* and *3* have a surplus of stock (PS_{2T}> 0, PS_{3T}> 0), the necessary quantity which makes it possible to eliminate the stock which is missing in depot *1*, for the first “Complte-Pooling” transshipment policy will be formulated by equation (7).

If $X_{21} \le PS_{2T}$ and $X_{31} \le PS_{3T}$ So $X_{31}=X_{21}= (D_{1T} - PS_{1T})$ **(7)**

If $X_{21}>PS_{2T}$ and $X_{31}>PS_{3T}$ and $X_{j1} \le (PS_{2T}+ PS_{3T})$ with j=2, 3.

So $X_{j1}= D_{1T} - PS_{1T}$

But for the last transhipment strategy “**Risk Balancing Strategy”** and precisely for the “Complete-Pooling” transhipment policy, the modeling by the ARENA 16.0 software can be presented in figure 5.

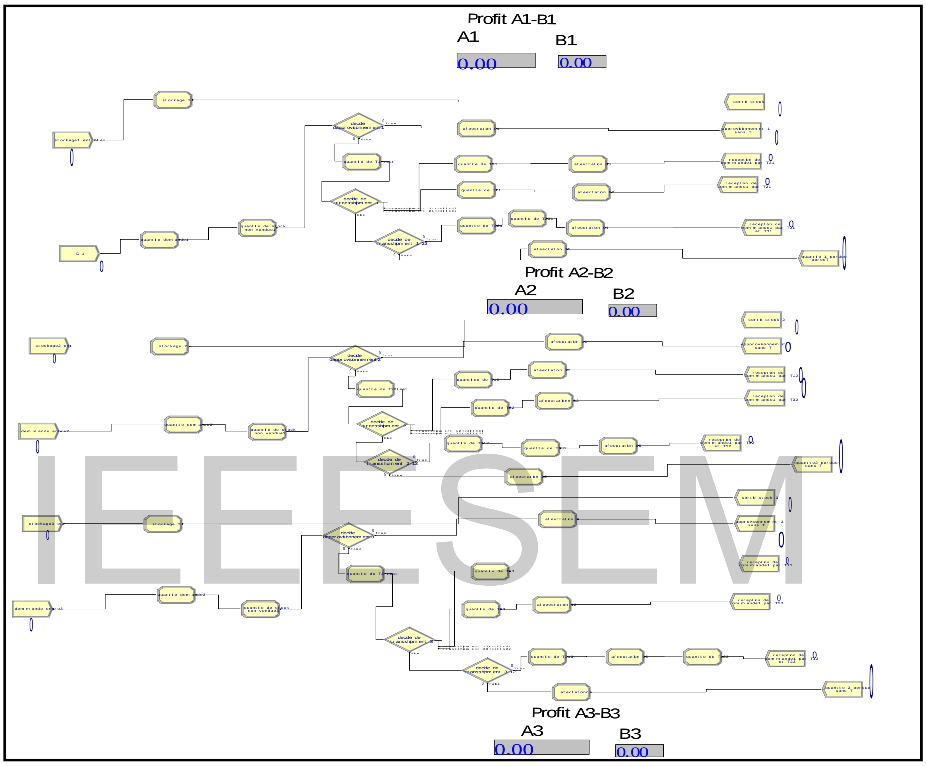

***Figure 5: the simulation model SC: Complete-Pooling for* Risk Balancing Strategy**

Whereas, for the second "Partial-Pooling" transshipment policy, this quantity transported laterally between retailers will be formulated using equation (8)

If $(PS_{2T} - \text{threshold}_{2T})>0$

And if $(PS_{3T} - \text{threshold})>0$

if $(D_{1T} - PS_{1T}) \le (PS_{2T} - \text{threshold}_{2T})$ So $X_{21}=D_{1T} - PS_{1T}$

if else et Si $(D_{1T} - PS_{1T}) \le (PS_{3T} - \text{threshold}_{3T})$ So $X_{31}=D_{1T} - PS_{1T}$

If else and if $(PS_{2T} - \text{threshold}_{2T})+(PS_{3T} - \text{threshold}_{3T}) \ge (D_{1T} - PS_{1T})$ **(8)**

So $X_{231}=D_{1T}-PS_{1T}$

Else $X11=(PS_{2T}-\text{threshold}_{2T})+(PS_{2T}-\text{threshold}_{2T})$

Else order lost

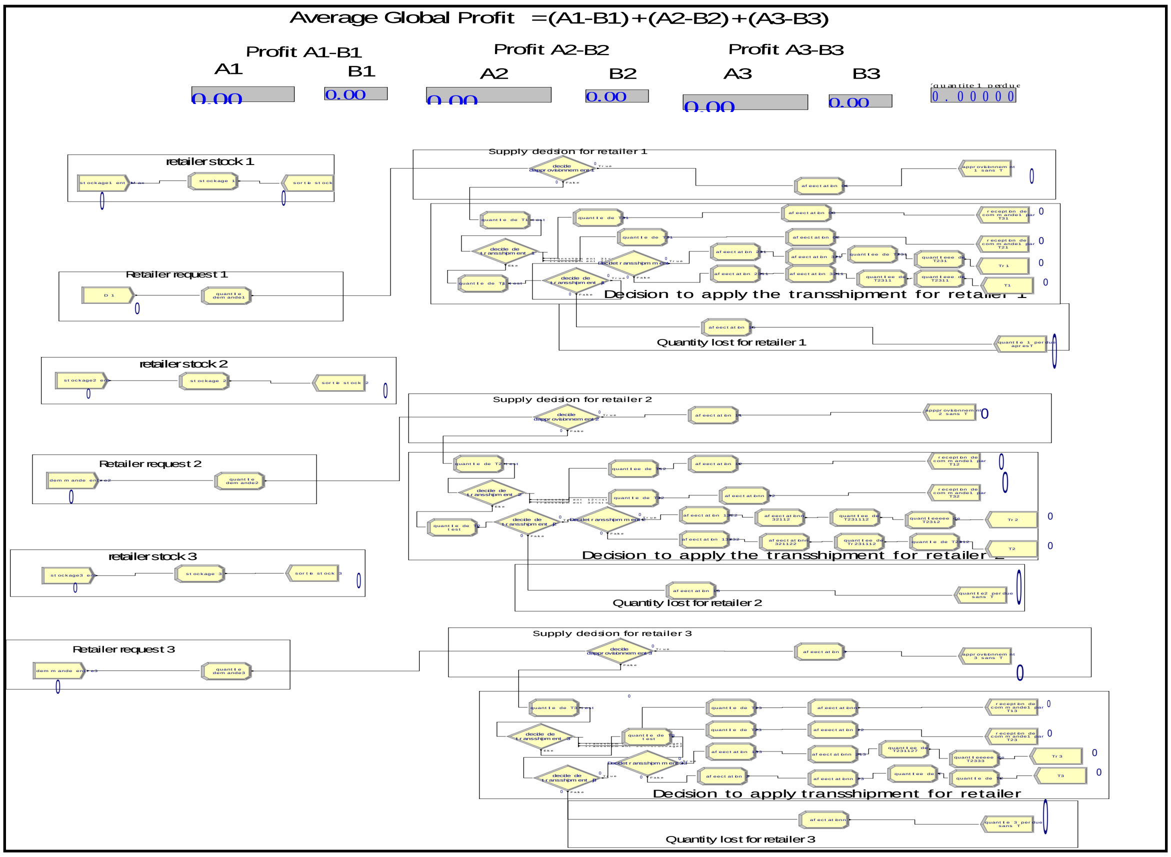

Figure 6: the simulation model SC: Partial-Pooling for **Risk Balancing Strategy**

3.2. Formulation of the problem:

3.2.1. Notations:

We adopt the following notations:

V_i: Unit selling price of site *i* with *i = 1, 2, 3,*

C_T: Unit cost of transshipment,

C_p : Unit cost of rupture whatever the site,

$E(D_{iT})$: Random average demand at each periodicity T, for each retailer i that follows the Normal law,

d_{12} , d_{21}:Distance separating the two storage sites 1 and 2, with $d_{12} = d_{21}$ (same thing for the other combinations of distance of the three retailers).

$E(I_i^+)$: Average quantity of residual stock after transshipment for retailer *i*,

$E(I_i^-)$: Average quantity of demand not satisfied according to site *i*, after the transshipment

$E(I_i)$:Average net stock at depot *i*, after transshipment

$E(X_i)$:Average quantity sold without transshipment from warehouse i,

PS_{iT} :Stock position at retailer i at the end of period T, with *i=1, 2, 3 and R=kT*

$E(GX_i)$: Average Global Quantity of transshipment received for retailer *i*, ∀*i=1, 2, 3,*

$E(TX_i)$: Average Global Quantity of transshipment sent according to retailer *i*, ∀*i=1, 2, 3,*

$\overline{TDG_i}$:Average GlobalDesservice rate for all retailers, it will be reformulated by the equation (9)

$$\overline{\mathbf{TDG_i}}=\mathrm{E}\left(\left(\sum_{i=1}^{3}\sum_{t=1}^{T}(I_i^-/D_i)\right)\right. \text{ , with R= kT et k=2, 3, 4,…, 10.} \quad \textbf{(9)}$$

$\overline{\Pi_i^G}$**(XG)** : Average Global Profit for retailers *i*,with *i=1, 2, 3.*

3.2.2. Global profit function without -Transshipment

The Average Global Profit without transshipment in a three-retailer inventory system with a linear transshipment cost includes the quantity sold without transshipment and the unfulfilled order quantity.

The mathematical formula of the Average Global Profit without transshipment for this stock system can be formulated by equation (10).

$$\overline{\Pi_i^G}(XG) = \mathrm{E}\left(\sum_{i=1}^{3}(V_i X_i - C_p I_i^-)\right) \quad \textbf{(10)}$$

3.2.3. Profit Function With -Transshipment

The Average Global Profit with integration of transshipment for a multi-retailer distribution system contains both the quantity supplied without the integration of transshipment, the quantity of transshipment transferred between retailers at the same level and the quantity lost.

The mathematical formula of the Average Global Profit for this storage system with the application of transshipment, for the "Complete-Pooling" transshipment policy will be formulated by equation (11).

$$\overline{\Pi_i^G}(XG) = \sum_{i=1}^{3}(V_i(\mathrm{E}(X_i) + \mathrm{E}(GX_i)) - C_T \mathrm{E}(TX_i) - C_p \mathrm{E}(I_i^-)) \quad \textbf{(11)}$$

3.2.4. Objective function

The objective is to improve the Average Global Profit of the distribution system over a finite time horizon R, composed of T periods. It includes the selling price, the unit cost of transshipment and the cost of rupture.
The mathematical formulation of the objective function, for the first "Complete-Pooling" transshipment policy, then takes the form of equation (12).

$$\mathrm{Max}\left(\sum_{i=1}^{3}(V_i(\mathrm{E}(X_i) + \mathrm{E}(GX_i)) - C_T \mathrm{E}(TX_i) - C_p \mathrm{E}(I_i^-))\right)$$

U/C

$$\begin{aligned} &X_{12} \le PS_{1T} \text{ and } X_{13} \le PS_{1T} \text{ With R= kT et k=2, 3, 4,…,10.}\\ &X_{21} \le PS_{2T} \text{ and } X_{23} \le PS_{2T} \text{ With R= kT et k=2, 3, 4,…,10.}\\ &X_{31} \le PS_{3T} \text{ and } X_{32} \le PS_{3T} \text{ With R= kT et k=2, 3, 4,…,10.}\end{aligned} \quad \textbf{(12)}$$

$S_i \geq 1$ Strictly positive integer, $\forall i = 1, 2, 3$

With

$S_i = (\mu_i * k + \sigma_i \sqrt{k})$, $\forall i = 1, 2, 3$ and with R= kT et k=2, 3, 4,…,10.

And

$X_i \sim N(\mu_i, \sigma_i)$, $\forall$ i = 1, 2, 3

While, for the second "Partial-Pooling" transshipment policy, the objective function will be defined in the form of equation (13).

$$\mathrm{Max}\left(\sum_{i=1}^{3}(V_i(\mathrm{E}(X_i) + \mathrm{E}(GX_i)) - C_T \mathrm{E}(TX_i) - C_p \mathrm{E}(I_i^- + X_{iii})\right)$$

U/C
$(PS_{iT} - \mathrm{threshold}_{iT}) > 0$ $\forall$ i = 1, 2, 3

(13)

With $\mathrm{threshold}_{iT}$ =Next Request; Twice the Request and 30% of PS_{iT}

And X_{iii} :The quantity lost from retailer i after stock accumulation with partial transhipment, $\forall i = 1, 2, 3$

$S_i \geq 1$ Strictly positive integer, $\forall i = 1, 2, 3$

With

$S_i = (\mu_i * k + \sigma_i \sqrt{k})$, $\forall$ i = 1, 2, 3 and with R= kT and k=2, 3, 4,…,10.

And

$X_i \sim N(\mu_i, \sigma_i)$, $\forall$ i = 1, 2, 3.

4. Characteristics of the methodology applied (Discrete Event Simulation)

Because of the limits of analytical resolution for certain aspects, remains complex and very difficult to solve. In particular, because the distribution of demand is random which makes the stock position for each retailer to be unknown and difficult to calculate, this leads us to resort to an approach by Discrete Event Simulation which we have given the possibility, at the same time, to relax the restrictive assumptions considered in the mathematical model and to analyze in a more detailed way the contributions of the transshipment and its sensitivity to different parameters (periodicity "T", threshold and unit cost of transshipment).
We describe, in the following section, the chosen resolution approach and the simulation model.
Besides, in our research work we assume that customer demand is a random variable, which leads to the application of the discrete event simulation approach. It consists of computer modeling by applying ARENA software, where the change in the state of a stock system over time is a series of discrete events. Each event (random demand) occurs at a given time and changes the state of the system.

Moreover, in this approach, we start by listing any events or state changes that may be encountered during the evolution of the inventory quantity. Then the logic of state changes is modeled in the form of algorithms by defining, for each type of event, the state conditions leading to the occurrence of the event as well as the corresponding state changes. The simulation of the stock system is obtained by executing the state change logics associated with each event on the date on which it occurs.

5. Analysis of the results

We recall that the structure of the network considered in this paper is composed of a distribution center and multi-retailers, which are faced with random requests which follow a Normal law of the mean μ and the standard deviation σ. These requests are independent and identically distributed *(i.i.d)*.
We then assume that :

- R = 28 days,
- C_T=1$, 3$,
- C_{pi}= 30$,
- V1=150$, V2=200$ et V3=170$,
- k= 2, 3, 4, 5, 6,7 ,8 ,9 ,10.

We solved our problem via the simulation approach by successively testing the effect of "Complete-Pooling" and "Partial-Pooling" transshipment policies on the Average Global Profit and Average Global Desservice Rate.

But for a number of retailers greater than two, it is necessary to choose the recipient and the recipient of the quantity transferred laterally between sites at the same level. This leads to a first study which focuses on choosing the best transshipment strategy in terms of economic profitability.

To select the best strategy, we considered the following performance measures for evaluating the contribution of the transshipment:

- The number of supply orders (without-transshipment),
- The number of orders fulfilled through transshipment,
- The quantity of transshipment transferred from a storage site which is in an overstock position to that of the same echelon which faces a rupture,
- The quantity of unsatisfied order at a retailer (quantity lost),
- The Average Global Profit at a retailer,
- The Average Global Desservice Rate.

5.1.Comparison of similarity of retailers

We assume that retailers face random demand that follows the Normal Law. But they are not the same in terms of the mean and standard deviation. For this we propose that the demand of the first retailer follows the law N (100,20), the second follows the law N (200,50) and the last follows the law N (150,30).
Table 1 presents an extract from various measurements of the initial level ofcompletely$S_i^0$for these different demands with *i = 1, 2, 3*.

Table 1: Determination of different measures of the initial level of Replenishment

k	S_1^0	S_2^0	S_3^0
2	229	470	343
3	335	687	501
4	440	900	660
5	545	1112	817
6	648	1322	973
7	753	1532	1130
8	857	1741	1285
9	960	1950	1440
10	1063	2158	1595

Numberspresented in Table 2 and which are calculated by simulation using ARENA software reveal the effect of similarity of retailers on ProfitGlobal Average.

Table 2: Determination of the Effect of Retailer Similarity on Profit Global Average

k	***Détaillants Similaires***					***Détaillants Non Similaires***				
	Without – transshipment	***Complete-Pooling***	***Partial-Pooling***			***Without-transshipment***	***Complete-Pooling***	***Partial-Pooling***		
			Twice the Demand	***Next Demand***	***SS=30% of PS_{iT}***			***Twice the Demand***	***Next Demand***	***SS=30% of PS_{iT}***
2	75670	80626	82697	93997	110076	72350	78350	79590	87367	93670
3	133770	162904	165585	175999	187915	130325	155657	157693	172750	179690
4	155521	170978	173539	225498	234468	151235	167695	168665	220567	229567
5	143871	152975	153200	197250	212000	133773	147690	148693	185630	207580
6	127177	135233	137527	180360	197725	120688	129369	130366	176900	189750
7	112483	128157	129330	165200	179650	109796	119255	112696	155650	170670
8	105790	113230	117520	157590	170600	100903	105360	107655	149950	162530
9	99097	107200	109330	142597	160750	99307	101569	103575	133695	155765
10	98403	102256	105530	130950	150500	973819	99690	100570	122670	140670

We notice from the analysis of Table 2 that the groups of retailers who face similar demands outperform from an economic point of view those who face non-similar demands and this is explicit in terms of relative improvement of the Average Global Profit of the "Without-Transshipment" policy and also with the application of "Transshipment".
For that we assume in what follows, that the random requests are identical and follow the law N (200,50).

5.2. Comparison between the different strategies in terms of Average Global Profit

5.2.1. Comparison between the two transshipment strategies: "Random transshipment strategy" and "Retailer proximity strategy"

Table 3 presents the different values of Average Global Profit for three storage sites located at the same level. First, we evaluated by simulation and using ARENA software these results without the application of transshipment between depots. Next, we study the possibility of existing cooperation between different retailers, and we seek to evaluate the performance of transshipment between warehouses on improving the economic profitability of the whole centralized system. This evaluation is done in the first place without taking into consideration any constraint, that is, by applying the "random transshipment strategy".

Table 3: Determination of the Average Global Profit according to the random transshipment strategy

k	***Average Global Profit***				
	Without-Transshipment	***Complete-Pooling***	***Partial-Pooling***		
			Twice the Demand	***Next Demand***	***SS=30% of PS_{iT}***
2	75670	80626	82697	93997	110076
3	133770	162904	165585	175999	187915
4	155521	170978	173539	225498	234468
5	143871	152975	153200	197250	212000
6	127177	135233	137527	180360	197725
7	112483	128157	129330	165200	179650
8	105790	113230	117520	157590	170600
9	99097	107200	109330	142597	160750
10	98403	102256	105530	130950	150500

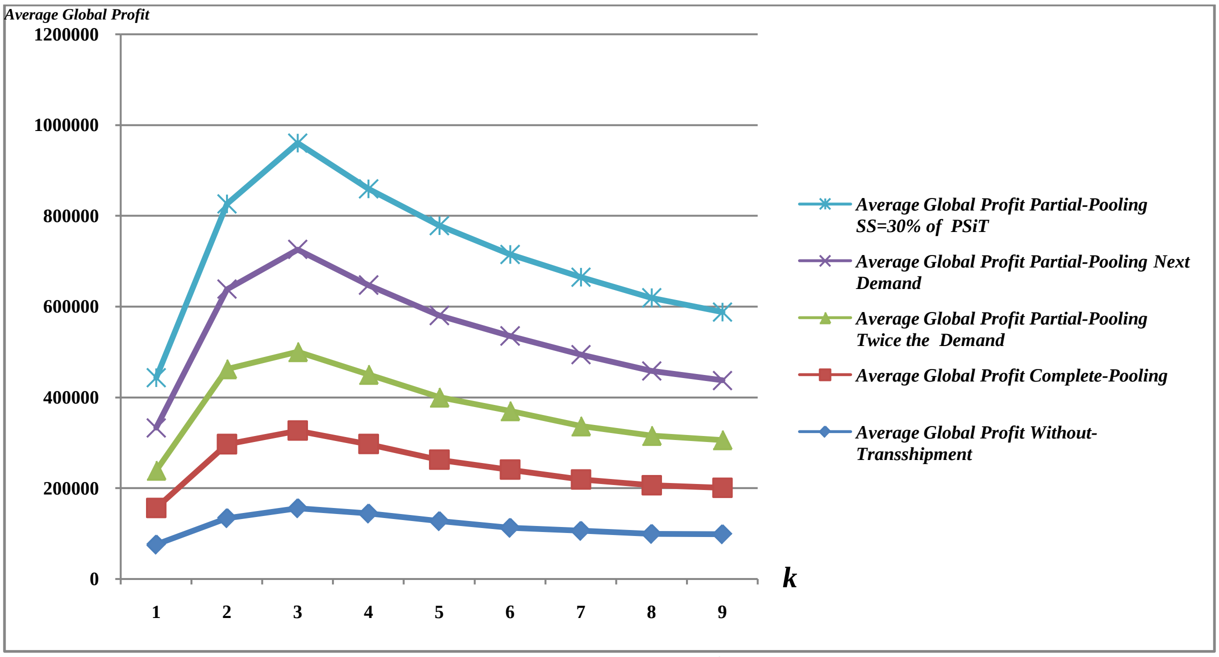

***Figure 7:* Average Global Profit**

We note, first of all, that these results verify those already obtained by the mathematical model of equations ((10) and (11)), for a stock system composed of three identical and independently distributed retailers (*i. i. d*).
By comparing the results of (Emel and Lena, 2017) where the number of collaborators is equal to two, with those found in this research for the same instances (the random requests are identical). It turns out that the Average Global Profit forecast by the sites in the group of two employees is lower than that forecast by the sites in the group of three. This means that the greater the number of sites in a group, the greater the gains, because breaking risk sharing is more effective when there are more sites sharing their inventories and there are more sources possible of lateral transfer in the event of an imminent out of stock..
From the table 3 and figure 7, we notice that the Average Global Profit has evolved by applying cooperation between retailers regardless of the periodicity. The percentage of relative improvement in the "Without-Transshipment" policy by applying cooperation between retailers ranging from 7% for k = 2 to 21% for k = 3 up to 10% for k = 4.
However, this improvement in profitability will be limited until the analysis of the effect of the distance constraint by applying the second transshipment strategy called "strategy according to the proximity of retailers".
For that, we require this constraint between the various retailers of the same level. So, we study all the possible combinations for these three sites (*1, 2* and *3*).

1^{st} Constraint **:** $d_{23}<d_{12}<d_{13}$

First, we require the second constraint by assuming that the distance between the two sites *2* and *3* is shorter than that between depot *1* and warehouse *2* and the latter is smaller than that between depots *1* and *3*.

Table 4: Determination of the Global Profit Average according to the retailer proximity strategy

k	***Without-Transshipment***	***Complete-Pooling***	***Partial-Pooling***		
			Twice the Demand	***Next Demand***	***SS=30% of PS_{iT}***
2	75670	83045	84577	98697	112277
3	133770	177780	179273	180933	195199
4	155521	180107	185765	230113	237165
5	143871	167675	172200	210750	221000

The table 4 summarizes the different results obtained by simulation by applying the first distance constraint.
In fact, the “Complete-Pooling” transshipment policy improves the Average Global Profit of the random transshipment strategy by a value of 3% for k equal to 2, to 9% for k equal to 3 and finally to 5 % for k equal to 4.
However, the “Partial-Pooling” policy acts on this improvement by varying the threshold for transshipment.

In fact,
- For a threshold = Twice the Demand: the “Partial-Pooling” transshipment policy improved the Average Global Profit of the random strategy by a relative improvement value equal to 2% for k equal to 2, then to 8 % for k equal to 3 and finally to 7% for *k* equal to 4.
- For a threshold = Next Demand: the average Global Profit of the previous strategy has improved by applying this threshold by a value equal to 5% for k equal to 2, then to 3% for k equal to 3 and finally to 2% for k equal to 4.
- For a threshold = 30% of PS_{iT}: the percentage of relative improvement of the previous strategy for a threshold equal to "30% of PSiT" is worth 2% for k equal to 2, to 4% for k equal to 3 and to 2% for k equal to 4.

From this table, we see that, for all the examples, the estimates calculated, with the application of the "Random Transshipment Strategy" have been increased by the integration of the first distance constraint. But the percentage of this improvement does not exceed 9%.

2^{th} *Contraint :* d_{13}<d_{12}<d_{23}

Next, we assume that, the distance separating the two sites 1 and 3 is shorter than that between depot 1 and warehouse 2 and the latter is narrower than that between depots 2 and 3.

Table 5: Determination of the Average Global Profit according to the "transshipment strategy according to the proximity of retailers"

k	*Without-transshipment*	*Complete-Pooling*	*Partial-Pooling*		
			Twice the Demand	*Next Demand*	*SS=30% dof* PS_{iT}
2	75670	82939	83997	972520	111980
3	133770	170665	171957	178115	193552
4	155521	181100	182997	228055	235978
5	143871	169205	165000	207957	210005

From table 5, we notice that the Average Global Profit of the first strategy (Random) with the integration of the second proximity constraint between the different retailers has been improved but also with low percentages and close to those of the first constraint. Indeed, through the application of the "Complete-Pooling" transshipment policy, this percentage never exceeds 6%, and also, with the second "Partial-Pooling" policy, this value reaches only 5%.

3^{Th} *Contraint :* d_{12}<d_{13}<d_{23}

After that, we propose that, the distance between the two sites 1 and 2 is shorter than that between the depot 1 and the warehouse 3 and the latter is shorter than that between the depots 2 and 3.

Table 6: Determination of the Average Global Profit according to the transshipment strategy according to the proximity of retailers

k	*Without-transshipment*	*Complete-Pooling*	*Partial-Pooling*		
			Twice the Demand	*Next Demand*	*SS=30% of* PS_{iT}
2	75670	85557	86517	95986	112277
3	133770	175785	177116	184519	194973
4	155521	176478	179097	223748	239380
5	143871	163205	165500	210750	230500

From the analysis of Table 6, we notice that the Average Global Profit of the random lateral transfer strategy, by applying this constraint of the distance tends to increase with a percentage improvement ranging from 3% to 8% for the “Complete-Pooling” policy while it reaches 7% for the second “Partial-Pooling” transshipment policy.

4^{th} *Contraint :* d_{23}<d_{13}<d_{12}

Then, we assume that, the distance between the two sites 2 and 3 is shorter than that between the depot 1 and the warehouse 3 and the latter is shorter than that between the depots 1 and 2.

Table 7: Determination of the Average Global Profit according to the "transshipment strategy according to the proximity of retailers"

k	*Without-transshipment*	*Complete-Pooling*	*Partial-Pooling*		
			Twice the Demand	*Next Demand*	*SS=30% of* PS_{iT}
2	75670	83295	85873	97697	111160
3	133770	174037	175271	186387	190573
4	155521	175946	180765	239773	243930
5	143871	162905	166700	213750	237700

According to Table 7, by integrating this constraint of proximity between retailers, we will conclude that the “Complete-Pooling” transshipment policy improves the Average Global Profit of the first transshipment strategy by a percentage ranging from 3% to 'at 7% while according to the use of the second transshipment policy, “Partial-Pooling” this relative improvement reaches 6%.

5^{th} *Contraint :* d_{12}<d_{23}<d_{13}

Then, we require that, the distance between the two sites 1 and 2 is more limited than that between the depot 2 and the warehouse 3 and the latter is narrower than that between the depots 1 and 3.

Table 8: Determination of the average Global Profit according to the "transshipment strategy according to the proximity of retailers

k	Without-transshipment	Complete-Pooling	Partial-Pooling		
			Twice the Demand	Next Demand	SS=30% of PS_{iT}
2	75670	86657	87382	98637	114750
3	133770	176057	178887	187770	199673
4	155521	179819	181897	240263	247782
5	143871	169907	173900	212950	240300

According to table 8, by requiring this constraint of proximity we find that, for the “Complete-Pooling” policy, the percentage of relative improvement of the Average Global Profit of the first transshipment strategy ranging from 5% to 8%. But, for the “Partial-Pooling” policy, this improvement also depends on the threshold of transshipment. Indeed for a threshold = Twice the Demand: this relative improvement in the Average Global Profit reaches 8%, then, for a threshold = Next Demandthe economic profitability of the first lateral transfer strategy undergoes an improvement of a percentage reached 7 %. And finally for a threshold = 30% of PSiT: the percentage of relative improvement in profitability also going up to 6%.

6^{th} *Contraint :* $d_{13}<d_{23}<d_{12}$

Finally, we require that, the distance between the two sites 1 and 3 is shorter than that between the depot 2 and the warehouse 3 and the latter is more limited than that between the depots 1 and 2.

Table 9: Determination of the Average Global Profit according to the "transshipment strategy according to the proximity of retailers"

k	Without-transshipment	Complete-Pooling	Partial-Pooling		
			Twice the Demand	Next Demand	SS=30% of PS_{iT}
2	75670	87670	88575	99876	115230
3	133770	177670	179281	189387	200270
4	155521	181527	182992	241269	248725
5	143871	170925	175350	213250	241522

By analyzing Table 9, and comparing it with the results of the random strategy, we notice that there is an improvement in the economic profitability of a percentage going to 9% for the “Complete-Pooling” transshipment policy, while that, for the second “Partial-Pooling” transshipment policy, it reaches 8%.

Note that, transshipment is generally significantly less expensive than an emergency order from a supplier if the side sites are located nearby. However, the benefits of transshipment should always be weighed against the costs involved.
It is for this reason, we conclude, whereas, the strategy of the transshipment "according to the proximity of the retailers" is more advantageous compared to the strategy of the random transfer in term of the economic profitability, but with a small percentage of improvement. . From this conclusion, we will study in the next section the comparison between the strategy of random transshipment and the “strategy of transshipment according to the risk confrontation”.

5.2.2.Comparison between the two strategies: "Random transshipment strategy" and "Transshipment strategy according to risk confrontation"

To make this comparison, we assume that in order to improve the fulfilled order quantity, we need to consider the risk of out of stock.

In fact, the strategy "according to the confrontationrisk "target to redistribute the stock among all the retailers to improve the Average Global Profit of the centralized system, through risk sharing between these collaborators.

Table 10 presents the different values found of the Average Global Profit by simulation using the ARENA software and applying the strategy "according to the risk confrontation".

Table 10: Determination of the Average Global Profit for the policies according to the “strategy of transshipment according to the risk confrontation”

k	Without-Transshipment	Complete-Pooling	Partial-Pooling		
			Twice the Demand	Next Demand	SS=30% of PS_{iT}
2	75670	105122	107537	120560	131222
3	133770	182187	185667	201675	207556
4	155521	246914	247646	255079	267059
5	143871	207325	209270	230350	259520

Interpreting this table, we notice that, whatever the transshipment policy applied, the Average global Profit of the "Random Transshipment Strategy" has tended to increase by sharing the risk between the different retailers.
In fact, by applying the “Complete-Pooling” transshipment policy, the Average Global Profit of the first strategy undergoes an increase whatever the periodicity T. Indeed, for k = 2, the relative improvement percentage is equal to 30%, for k = 3 it will be equal to 12% and for k = 4 it becomes 45%.

While, by using the second transshipment policy, "Partial-Pooling", the Average Global Profit of the first strategy has been improved according to the threshold level. To do this, first of all, with the setting of the threshold at Twice the Demand, the latter increased with a relative improvement percentage ranging from 12% for k equal to 3 up to 43% for k equal to 4. Then , for a threshold fixing to the Next Demand, the economic profitability at all the retailers tended to improve by a percentage of 28% for k = 2, by 15% for k equal to 3 and by 13% for k equal to 4. Finally, with a threshold equal to 30% of stock position, the relative improvement percentage will be equal to 20% for k = 2, to 11% for k = 3 and to 14% for k = 4.

We will then conclude, from the values found in Table 10, that the "risk confrontation" strategy is more advantageous over the previous strategy because it makes the centralized system more profitable.

This comparison leads to the conclusion that the strategy of lateral transfer of product between sites (random, according to the level of risk or according to the proximity of retailers) influences the performance of the centralized stock system.

5.3. Comparison between the different strategies in terms of Average Global Desservice Rate

5.3.1.Comparison between the two transshipment strategies: "random" and "depending on the proximity of retailers"

First, we will take into consideration the application of the "random transshipment strategy" to find the different values of the Average Global Desservice Rate.

Table 11 shows these different values for three depots located at the same "Without-Transshipment" level and “with-Transshipment" application, using the ARENA simulation software.

To calculate these values, we apply formula (11) which designates the Average Global Desservice Rate.

$$\overline{\mathbf{TDG}}_i = \boldsymbol{E}((\textstyle\sum_{\mathrm{i}=1}^{3}(\boldsymbol{I}_i^-/\mathbf{D}_\mathrm{i})) \qquad (11)$$

Table 11: Determination of the Average Global Desservice Rate according to the "random transshipment strategy"

k	***Wuthout-transshipment***	***Complete-Pooling***	***Partial-Pooling***		
			Twice the Demand	***Next Demand***	***SS=30% of*** PS_{iT}
2	0.870	0.760	0.730	0.550	0.470
3	0.930	0.581	0.565	0.423	0.395
4	0.850	0.476	0.463	0.367	0.342
5	1.200	0.975	0.907	0.520	0.425
6	1.392	1.075	1.005	0.792	0.575
7	1.466	1.252	1.200	0.920	0.782
8	1.657	1.590	1.505	1.023	0.902
9	1.852	1.697	1.606	1.262	1.003
10	1.977	1.875	1.807	1.559	1.000

We have studied the effect of transshipment policies on the minimization of the Average Global Desservice rate in a centralized system composed of multi-retailers. First of all, we calculated the rate of the quantity of order lost for the case of “without-transshipment”, then we looked for it with the integration of “Transshipment” by the use of these two policies by exploring the impact of each on the variation in the Average Global Desservice rate.

From table 11, we notice that, whatever the periodicity T, the “Complete-Pooling” transshipment policy makes it possible to minimize the rate of the quantity lost by the collaboration between the depots in the event of a out of stock.

In fact,

➢ For k = 2: the Average Global Desservice Rate tended to decrease from 0.870 to 0.760,

➢ For k = 3: this reduction is equal to 0.930 to 0.581,

➢ For k = 4: this rate decreases from 0.850 to 0.476,

While, with the application of the second "Partial-Pooling" transshipment policy, this rate will be reduced and the decline value depends on the setting of the threshold.

Then, for a threshold equal to “Twice the Demand”: the average lost quantity of the entire system decreases by contribution to that found by applying the “Complete-Pooling” transshipment policy and this according to the variation of the periodicity.

In fact,

- For k = 2: this decrease 0.760 to 0.730,
- For k = 3: this reduction is worth 0.581 to 0.565,
- For k = 4: this rate decreases from 0.476 to 0.463.

Also, for a threshold equal to the “Next Demand” and regardless of the periodicity T, the quantity of unsatisfied customer orders has been reduced. Indeed, for k = 2, it decreases from 0.730 to 0.550, for k = 3 from 0.565 to 0.423 and finally for k = 4 from 0.463 to 0.367.

Likewise, for a threshold equal to 30% of PSiT: the rate of non-satisfaction of customer demand has decreased regardless of the frequency.

We notice that, by applying the “Without-Transshipment” and “With -Transshipment” policy, whatever the transshipment policy applied (Complete-Pooling or Partial-Pooling), beyond k = 4, the Desservice Rate Global Average will be presented as an increasing curve.

Second, we will take into consideration the application of the "retailer proximity" transshipment strategy to find the different values of the Average Global Desservice Rate after the distance constraint requirement.

$\mathbf{1}^{st}$ Constraint **:** $\boldsymbol{d_{23}}$<$\boldsymbol{d_{12}}$<$\boldsymbol{d_{13}}$

First, we assume that the distance between the two sites 2 and 3 is shorter than that between depot 1 and warehouse 2 and the latter is narrower than that between depots 1 and 3.

Table 12: Determination of the Average Global Desservice Rate "strategy according to proximity to retailers"

k	***Without-transshipment***	***Complete-Pooling***	***Partial-Pooling***

			Twice the Demand	*Next Demand*	*SS=30% of PS_{iT}*
2	0.870	0.610	0.597	0.313	0.205
3	0.930	0.403	0.387	0.297	0.182
4	0.850	0.390	0.370	0.230	0.157
5	1.200	0.775	0.758	0.350	0.272

Based on these results, regardless of the transshipment policy applied, the Global Average Customer Order Non-Satisfaction Rate decreases because of the geographic proximity between retailers.

In fact, by applying the first policy, "Complete-Pooling", for k = 2, the lost order rate undergoes a decrease from 0.760 to 0.610, for k = 3, this decrease going from 0.581 to 0.403 and for k = 4, this rate decreases from 0.476 to 0.390.

Whereas, for the “Partial-Pooling” transshipment policy, this decrease in the quantity of the unsatisfied order does not depend only on the periodicity T but also on the variation of the transshipment threshold and this is explicit according to the values presented in Table 13, taking as an example, for k = 4, for a threshold equal to "Twice the Request" the rate of the quantity lost equal to 0.370, but for a threshold equal to "the Next Request," this rate decreases to 0.230, while for the last threshold which is equal to “30% of PS_{iT}”, it will be equal to 0.157.

From Table 12, we will conclude that, the strategy of "retailer proximity transshipment" with the requirement of the first distance constraint improves the results found by the random transshipment strategy and this results in the increase of the “Average Global Profit” calculated in the previous section.

2^{th} *Contraint :* $d_{13}<d_{12}<d_{23}$

Next, we require that the distance between the two depots 1 and 3 be narrower than the distance between depot 1 and warehouse 2 and that the latter be tighter than the distance between storage sites 2 and 3.

Table 13 summarizes the different results obtained by simulation by requiring the second distance constraint.

Table 13: Determination of the rate of Average Global Desservice according to the “retailer proximity strategy”

k	*Without-transshipment*	*Complete-Pooling*	*Partial-Pooling*		
			Twice the Demand	*Next Demand*	*SS=30% of PS_{iT}*
2	0.870	0.697	0.689	0.407	0.300
3	0.930	0.403	0.400	0.375	0.257
4	0.850	0.380	0.375	0.300	0.197
5	1.200	0.823	0.800	0.496	0.395

From the figures calculated in Table 13, and by comparing them with those found for the first distance constraint, we first conclude that, for the “Complete-Pooling” transshipment policy, the quantity of orders lost from the whole centralized system undergoes a slight growth from 0.610 to 0.697 for k = 2 and this results in the weak decrease of the Average Global Profit found in the previous part of the paper. While, for the second periodicity (k = 3), this Average GlobalDesservice rate does not vary, but the cause of the decrease in economic profitability is that retailers 3 apply the transaction of transshipment with retailer 2 more than with retailer 1 because the latter's stock position is not able to meet the demand of 3 while the distance between it is shorter.

But, for k = 4, the unfulfilled order quantity decreases and acts positively on the improvement of the average global profit of the whole centralized system.

Then, for the “Partial-Pooling” transshipment policy, the value of the Average Global Desservice Rate undergoes an increase regardless of the threshold beyond which the transshipment is applied as a stock competitor, which results in a decrease in the Global gain of centralized system.

3^{th} *Contraint :* $d_{12}<d_{13}<d_{23}$

Then, we require that, the distance between the two sites 1 and 2 is shorter than that between the depot 1 and the warehouse 3 and the latter be narrower than the one between the depots 2 and 3.

Table 14: Determination of the Average Global Desservice rate according to the "transshipment strategy according to the proximity of retailers"

k	*Without-transshipment*	*Complete-Pooling*	*Partial-Pooling*		
			Twice the Demand	*Next Demand*	*SS=30% of PS_{iT}*
2	0.870	0.520	0.503	0.453	0.283
3	0.930	0.397	0.380	0.296	0.200
4	0.850	0.380	0.378	0.313	0.117
5	1.200	0.700	0.697	0.379	0.279

We will look for the effect of changing the distance constraint on the decrease in the Average GlobalDesservice Rate for a centralized system. For this reason, we will study the third constraint and by comparing it with the previous one, we note first of all that, according to the “Complete-Pooling” transshipment policy, this rate decreases for the first two periods and this leads to a increase in system profitability by a percentage equal to 3% while, for k = 4, this rate does not change but the cause of the decrease in the average global profit is that, retailer 1 performs the operation transshipment with retailer 3 more than with retailer 2.

Next, we will study the effect of the second transshipment policy on the unfulfilled order quantity. In fact, first of all, for the first transshipment threshold set at “Twice the Demand”, the Average Global Desservice Rate undergoes a decrease for the first two intervals which results in a slight improvement in the Average Global Profit.

While for the last periodicity, it increases and this influences the degradation of the economic profitability of the whole centralized system.

Second, for the threshold equal to “Next Request”, this Desservice rate increases for k = 2 and k = 4, which results in a degradation of the average Global profit. Whereas, for k = 3, this rate undergoes a decrease which influences the improvement of economic profitability.
Finally, for a threshold set at "30% PSiT", this rate undergoes a slight decrease which results in a slight increase in the Average Global Profit, with a relative improvement percentage never exceeding 1%.

4^{th} *Contraint :* $d_{23}<d_{13}<d_{12}$

Then, we require that the distance between the two sites 2 and 3 be more limited than that between the depot 1 and the warehouse 3 and that the latter be shorter than the one between the depots 1 and 2.

Table 15: Determination of the Average Global Desservice Rate according to the "transshipment strategy according to the proximity of retailers"

k	***Without-transshipment***	***Complete-Pooling***	***Partial-Pooling***		
			Twice the Demand	***Next Demand***	***SS=30% of*** PS_{iT}
2	0.870	0.550	0.527	0.409	0.325
3	0.930	0.397	0.380	0.205	0.200
4	0.850	0.380	0.360	0.190	0.100
5	1.200	0.715	0.597	0.220	0.262

According to the analysis of the figures presented in table 4.15 and after the comparison with those of table 14, we note first of all that, the Average Global Desservice Rate for the “Complete-Pooling” policy, underwent an increase for the first periodicity which results in a decrease in the Average Global Profit with a degradation percentage equal to 2%. But, for the second and third periodicity, it does not change and the cause of the decrease in the average Global profit is that, retailer 2 transfers a large amount of transshipment to retailer 1 which is located far away from retailer 3. .
Then, we notice that, for the “Partial-Pooling” transshipment policy, and by setting the threshold at “Twice the demand”, this rate undergoes a growth for the first and periodicity, therefore this results in a degradation of the profitability of the the whole system. Whereas, for k = 3, this rate does not modify, and the main cause of the degradation of the economic profitability of the whole centralized system, is that, retailer 3 transfers a large quantity to the retailer who is located furthest away. . Finally, for k = 4, this rate will be reduced and this leads to an improvement in economic profitability.
Then, by setting the threshold at the "Next Demand", this rate decreases regardless of the frequency and this positively influences the improvement in average oglobal profit.
Finally, by analyzing the last threshold (equal to "30% of PS_{iT}"), we find that for the first periodicity, this rate undergoes a weak growth, which leads to a slight deterioration in the profitability of the system. But, for the second, it does not vary, and the main source of this decrease in the Global gain of the three retailers is that, the amount of transshipment transferred between retailers 1 and 2 is very large. While, for the last periodicity, it will be reduced and this results in an improvement in profitability.

5^{th} *Contraint :* $d_{12}<d_{23}<d_{13}$

After that, we assume that, the distance between the two sites 1 and 2 is shorter than the one between the warehouse 2 and the warehouse 3 and that the latter is narrower than the one between the depots 1 and 3.

Table 16: Determination of the Average Global Desservice Rate according to the "transshipment strategy according to the proximity of retailers"

k	***Without-transshipment***	***Complete-Pooling***	***Partial-Pooling***		
			Twice the Demand	***Next Demand***	***SS=30% of*** PS_{iT}
2	0.870	0.530	0.503	0.409	0.220
3	0.930	0.382	0.377	0.187	0.170
4	0.850	0.379	0.350	0.175	0.090
5	1.200	0.695	0.526	0.207	0.228

According to the study of Table 16 and after the comparison of these values calculated by the ARENA software with those of Table 15, we first notice that, for the “Complete-Pooling” transshipment policy, the rate of unfulfilled order quantity throughout the system suffered such a decrease regardless of the periodicity, resulting in an increase in the Average Global Profit.
Then, for the “Partial-Pooling” transshipment policy and by setting the threshold at “Twice the Demand”, we notice, whatever the periodicity, the Average Global Desservice Rate undergoes a slight decrease and this has a slight influence on l 'improvement of the global economic profitability of the system.
Then, by setting the threshold to "Next demand", the rate of the unsatisfied quantity does not vary for the first periodicity, but the essential cause of the improvement in the average Global profit is the large quantity transferred between retailers 1 and 2. While, from the analysis of the other two periodicities, we find that, the quantity lost undergoes such a decrease and this resulted in an improvement of the Global profit of the whole centralized system.
Finally, by setting the threshold at "30% of PS_{iT}", the quantity in failure of the whole system will be reduced whatever the periodicity which positively influences the profitability of the system.

6^{th} *Contraint :* $d_{13}<d_{23}<d_{12}$

Finally, we assume that, the distance between the two sites 1 and 3 is more limited than that between the deposit 2 and the site 3 and the latter is shorter than that between the deposits 1 and 2.

Table 17: Determination of the Average Global Desservice Rate according to the “transshipment strategy according to the proximity of retailers”

k	*Without-transshipment*	*Complete-Pooling*	*Partial-Pooling*		
			Twice the Demand	*Next Demand*	*SS=30% of PS_{iT}*
2	0.870	0.497	0.470	0.375	0.197
3	0.930	0.365	0.357	0.165	0.120
4	0.850	0.350	0.340	0.153	0.080
5	1.200	0.560	0.493	0.193	0.179

From the figures calculated in table 17, and by comparing them with those found in table 16, we will first conclude that, for the "Complete-Pooling" policy, and whatever the periodicity, the rate of Average Global Desservice is decreasing and this results in an increase in Average Global Profit for the three retailers. Then, for the second policy of the “Partial-Pooling” lateral transfer, and by first setting the threshold at “Twice the Demand”, we notice that, whatever the periodicity, the Average Global Profit increases while The Average Global Desservice Rate is decreasing, and the reason for improving the economic profitability of the whole centralized system is that Retailer 1 is doing lateral transfer with 3 more than Retailer 2.

Then, by fixing it to the “Next Demand”, and to the “30% of PS_{iT}”, we find that, the Average Global Profit undergoes an improvement because of the decrease in number of unfulfilled orders.

According to the application of the proximity of the distance between the retailers, we will conclude that the more the latter are close this leads to improve the cooperative relationship between its different sites, but it is necessary to balance the quantities transferred laterally between them to increase as much as possible the economic profitability of a centralized system.

Until recently, the main limitation in applying this strategy for finished goods and consumables was insufficient and unreliable information on current stocks at each site. Thus, information should be shared between different retailers and made more reliable.

This information could be used by lateral transshipments not only to satisfy the demand of customers who are willing to wait, but also to predict stockouts, i.e. proactive transshipments.

An interesting observation that we can draw from this first comparison is that the “Retailer proximity strategy” is more profitable than the random strategy, but with a low margin. From this conclusion, we will study in the next section the comparison between the strategy of random transshipment and “the strategy of transshipment according to the risk confrontation”.

5.3.2.Comparison between the two transshipment strategies: "random" and "according to the risk confrontation"

In order for the Average Global Service rate to have a strong relationship with the relative improvement in the Average Global Profit, we will look for the lateral transfer strategy and the transshipment policy that aims to minimize this unsatisfied quantity as much as possible.

Table 18: Determination of the Average Global Desservice Rate for according to the "transshipment strategy according to the risk confrontation"

k	*Without-transshipment*	*Complete-Pooling*	*Partial-Pooling*		
			Twice the Demand	*Next Demand*	*SS=30% of PS_{iT}*
2	0.870	0.290	0.277	0.095	0.076
3	0.930	0.273	0.269	0.072	0.053
4	0.850	0.220	0.215	0.030	0.003
5	1.200	0.300	0.287	0.095	0.060

In fact, from the figures found in Table 18, and after their comparison with those calculated for the random transshipment strategy, we will first conclude that, for the first “Complete-Pooling” transshipment policy, this rate decreases regardless of the periodicity T. Indeed, for k = 2, this decrease in the Average Global Desservice rate is from 0.870 to 0.290, for k = 3, this decrease is from 0.930 to 0.273 and for k = 4, this rate decreases from 0.850 to 0.220. This results in improved economic profitability by increasing the Average Global Profit of the random strategy by a relative improvement percentage value of 45%.

Then, for the "Partial-Pooling" transshipment policy, whatever the threshold, we notice that this rate decreases until it reaches values close to zero (optimal value of the Average Global Desservice rate). And this is because of the efficient redistribution of the quantity transferred laterally between retailers.

We then conclude that the strategy “according to the risk confrontation” is more advantageous compared to the strategy “according to the proximity of the retailers” in terms of the Average Global Profit and also of the Average Global Desservice Rate with a large margin, because in supply chain, inventories are inevitable realities due to the unpredictable uncertainties of the operating process. The risk confrontation strategy was designed to create an aggregation of demand across sites or over time.

As demand levels vary from retailer to retailer, high demand from one location tends to be offset by low demand from another. This reduction in variability allows a decrease in average stock, thus increasing the expected Average Global Profit of the system. For this, in supply chain management, “Risk-Pooling” generally becomes more efficient for a centralized system with an aggregated inventory in a distribution center, instead of a decentralized system with a separate inventory.

From this second conclusion, all future comparisons and discussions will be based on this strategy.

5.4. Effects of input parameters on "Average Global Profit" and "Average Global Desservice Rate" for "Transshipment strategy according to risk confrontation"

We also identify the input parameters that act on the profit of the transshipment improving the Average Global Profit by minimizing the Average GlobalDesservice Rate namely, the standard deviation and the average demand per period.

5.4.1. Effects of standard deviation($\sigma_1 = \sigma_2 = \sigma_3 = \sigma$)

We examine, first, in an identical retailer system, the impact of changing the standard deviation **σ** from 50 to 20 and then to 75 with a fixation of the average demand per period **μ** at 200 units.

Table 19: Determination of the different values of the level of Recompletion for different **σ**

k	***Replenishment Level*** S_i^0		
	N(200,50)	**N(200,20)**	**N(200,75)**
2	470	430	507
3	687	635	730
4	900	840	950

We study, for the "Complete-Pooling" and "Partial-Pooling" transshipment policies, the effect of the demand standard deviation on the performance in the standard deviation identical retailer inventory system.

Table 20: Determination of the Average Global Profit in $ for "transshipment strategy according to the risk confrontation" according to the variation of σ.

k	***Complete-Pooling***			***Partial-Pooling***								
				Twice the Demand			***Next Demand***			***SS=30% of*** PS_{iT}		
	σ			**σ**			**σ**			**σ**		
	50	20	75	50	20	75	50	20	75	50	20	75
2	105122	103235	107552	107537	106115	110190	120560	118007	123388	131222	129288	132565
3	182187	178592	187870	185667	181285	189359	201675	197289	203421	207556	205825	209675
4	246914	241127	247669	247646	245937	248292	255079	246697	257825	267059	263277	269008

Table 21: Determination of the Average Global Desservice rate for "transshipment strategy according to the risk confrontation" according to the variation of σ

k	***Complete-Pooling***			***Partial-Pooiling***								
				Twice the Demand			***Next Demand***			***SS=30% of*** PS_{iT}		
	σ			**σ**			**σ**			**σ**		
	50	20	75	50	20	75	50	20	75	50	20	75
2	0.290	0.317	0.189	0.277	0.329	0.173	0.098	0.109	0.077	0.076	0.086	0.050
3	0.273	0.302	0.165	0.269	0.309	0.149	0.072	0.095	0.065	0.053	0.066	0.037
4	0.220	0.297	0.151	0.215	0.292	0.132	0.030	0.077	0.021	0.003	0.027	0.001

The simulation results presented in Tables 20 and 21 show that a variation in the standard deviation of demand acts mainly on the variation in Average Global Profit and Average Global Servicing Rate.

For this reason we find that, passing from σ = 50 to σ = 20, the Average Global Profit undergoes a decrease and the unsatisfied quantity increases and the reverse for the passage from σ = 50 to σ = 75.

First of all, we study the influence of passage from σ = 50 to σ = 20, It is remarkable that by applying the policy of transshipment "Complete-Pooling", this variation of the Average Global Profit and the quantity in rupture has a relation to the change of the periodicity T.

Indeed,

- For k = 2: the Average Global Profit decreases by 1% compared to that with the standard deviation σ = 50, but, the Average Global Desservice Rate has increased by a value equal to 0.290 to 0.317.
- For k = 3: the average global profit decreases by 2% compared to that with the standard deviation σ = 50, but, the Average Global Desservice Rate has increased from 0.273 to 0.302.
- For k = 4: the Average Global Profit decreases by a value equal to 2% from σ = 50 to σ = 20, but, the Average Global Desservice Rate has evolved from 0.220 to 0.297.

Whereas, for the "Partial-Pooling" policy, and whatever the transshipment threshold, the quantity lost increases and the economic profitability of the whole centralized system decreases with the reduction of the standard deviation from 50 to 20.

Then, we study the influence of passage from σ = 50 to σ = 75, we notice from the figures calculated in the two tables 4.20 and 4.21 that an increase in the variation of demand σ from 50 to 75, results in a significant growth in the value of economic profitability by improving the

Average Global Profit of the centralized system and by reducing the Average Global Desservice Rate and this is explicit for the two transshipment policies "Complete-Pooling" and "Partial-Pooling" which whatever the threshold of transshipment. Very reassuring to note is that increasing the standard deviation gives a net advantage to policies with transshipment by reducing σ from 50 to 75.

5.4.2.Effects of average demand by period ($\mu_1=\mu_2=\mu_3=\mu$)

Next, we study the impact of the change in average demand per period μ of the inventory system at identical retailers is performed in the case where σ = 50.

A summary of different values of average global profit and the Desservice rate is extracted in tables 22 and 23.

This is done to study the behavior of "Complete-Pooling" and "Partial-Pooling" policies with different transshipment thresholds with the variation in average demand per period, on the performance of the storage system.

By taking, in the first case, the passage from μ = 200 to μ = 100, then in the second case from μ = 200 to μ = 300.

Table 22: Determination of the different replenishment level values for different μ

k	***Replenishment Level*** S_i^0		
	N(200,50)	**N(100,50)**	**N(300,50)**
2	470	270	670
3	687	387	987
4	900	500	1300

Table 23: Determination of the average global profit in $ for "" Transshipment strategy according to the risk confrontation "according to the variation of μ

k	***Complete-Pooling***			***Partial-Pooling***								
				Twice the Demand			***Next Demand***			***SS=30% of*** PS_{iT}		
	μ			μ			μ			μ		
	200	100	300	200	100	300	200	100	300	200	100	300
2	105122	109999	103405	107537	109025	105256	120560	123690	117925	131222	132000	129925
3	182187	185375	180832	185667	187705	180205	201675	203719	198791	207556	209375	206871
4	246914	246851	244821	247646	249825	245895	255079	260965	250255	267059	269440	265728

Table 24: Determination of the average global desservice rate for "transshipment strategy according to the risk confrontation" according to the variation of μ

k	***Complete-Pooling***			***Partial-Pooling***								
				Twice the Demand			***Next Demand***			***SS=30% of*** PS_{iT}		
	μ			μ			μ			μ		
	200	100	300	200	100	300	200	100	300	200	100	300
2	0.290	0.197	0.307	0.277	0.163	0.297	0.095	0.077	0.117	0.076	0.057	0.097
3	0.273	0.157	0.287	0.269	0.127	0.273	0.072	0.063	0.113	0.053	0.046	0.068
4	0.220	0.141	0.293	0.215	0.109	0.235	0.030	0.022	0.092	0.003	0.002	0.035

The performance sensitivity analysis to the variation in average demand per period (μ) is summarized as follows (See tables 23 and 24):

The lower the average demand per period (μ), the more the Average Global Profit increases, which obviously leads to a decrease in terms of the Average Global Desservice Rate, whatever the transshipment policy and whatever the periodicity T.

We conclude then that an increase in the average demand per period (μ) results in a decrease in the relation of the transshipment between the storage sites of the same grade and consequently, each retailer takes precautions to reduce the quantity of disruption by reserving more of stocks. This results in a decrease in terms of the Average Global Profit and an increase in terms of the Average Global Desservice Rate.

We can also conclude that this paper has examined the relevant factors that can influence the behavior of the system (the variance σ, the standard deviation, the threshold of transshipment). A summary of the important points of this analysis is as follows:

- The results presented in this study show that sharing inventory between sites always leads to significant improvements in the average Global profit of the system;

- The sharing of stocks between sites guarantees a high level of service for customers (less shortage) and more economical in terms of costs;

- In the event that the cost of lateral transfer is very high, it is not advisable to adopt the strategy of collaboration between sites.

6. Conclusion

We have first of all dealt with in this paper the problem of transshipment to several retailers ($n \geq 3$) in a two-tiered distribution network, for this, we conducted an empirical study by studying three different transshipment strategies (random, depending on the proximity of retailers, and depending on the risk confrontation.

Then, we study the effectiveness of different strategies on the reduction of rupture risk and we conclude that, the strategy of transshipment according to the risk confrontation is the most economically profitable.

Finally, we set out to measure the impact of the input parameters on the benefit of transshipment policies, “Complete-Pooling” and “Partial-Pooling” for the transshipment strategy according to the risk confrontation and we note that an increase in terms of average demand leads to a deterioration in economic profitability by reducing the Average Global Profit and increasing the Average Global Desservice Rate. While an increase in terms of uncertainty (standard deviation) acts positively on improving the economic profitability of the whole centralized system.

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