

Mean To The Empirical Departures Distribution

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Abstract

This paper presents extended statistical properties for one of the new empirical distribution by using general known familiar methods.

Keywords: Random variable; continuous probability distribution; departure rate; density function; mean of the distribution.

Introduction

Statistics has the most interesting solutions for the problems in several fields due to its universality. Several new distributions have been developed by taking some subtle transformations on the existing distributions. This paper is continuous work to the previous one [1] which is briefed here. The Random variable of interest is to “how likely there are successive departures in a particular interval”. Instead of asking “how many departures take place in a particular time interval (Poisson)”, we ask for “how likely there are successive departures in a particular interval”. Since X is continuous, the PDF should be a function. We had made some inferences about this unknown function. This means the probability distribution that takes into account of measurements those we have surveyed for a considerable period of time. So the output of the inference problem is the distributions of X. We charted the histograms for successive arrivals in a particular interval from which we found the density curves. [1][2]

In which case its probability density function is given by

$$f(x) = \begin{cases} 2\mathfrak{Z}'P(n, x) - P(n-1, x) & \text{when } t \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad [1]$$

where $P(n, t)$ is the Truncated Poisson probability distribution of remaining n customers in the queuing system after departing $N-n$ customers from the queuing system in time interval $[0, t]$. [1]

$P(n-1, t)$ is the Truncated Poisson probability distribution of remaining $n-1$ customers in the queuing system after departing $N-(n-1)$ customers from the system in time interval $[0, t]$ and \mathfrak{Z}' is the normalizing constant. [1]

In recent years, several methods for generating new models for classic distributions have been proposed. A detailed study about “the evolution of methods for generalizing classic distributions” was made by Lee et al. [4]. The objective of this study is to study statistical features of the above distribution. Thus this paper presents the mean of the above new empirical distribution based on the general known routine techniques.

Graphs of the density function:

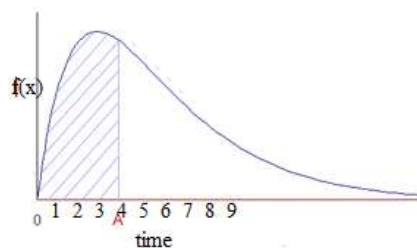


Fig. 1. Density function 1

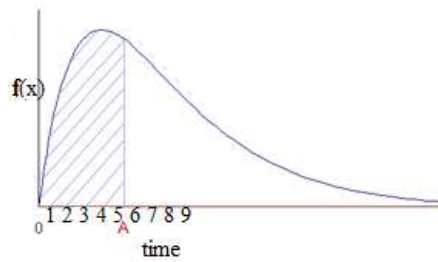


Fig. 2. Density function 2

To extend this distribution theory, the cumulative distributive function and interesting properties of all statistical distributions mean, variance and standard deviation are studied which are widely used in several fields insurance, management, business and finance etc. [1]

Mean of the Distribution

$$E(x) = \int_0^{\infty} xf(x)dx \quad [3]$$

When $n = 2$

$$\int_0^{\infty} x\mu [2\tau^1 P(n, x) - P(n-1, x)] dx$$

$$E(x) = \int_0^{\infty} x\mu [2\tau^1 P(2, x) - P(1, x)] dx$$

$$= \int_0^{\infty} \mu x 2\tau^1 \left(\frac{e^{-\mu x} (\mu x)^2}{2} \right) dx - \int_0^{\infty} x\mu e^{-\mu x} \mu x dx$$

$$= \int_0^{\infty} \tau^1 e^{-\mu x} (\mu x)^3 dx - \int_0^{\infty} e^{-\mu x} (\mu x)^2 dx$$

$$\frac{6\tau^1}{\mu} - \frac{2}{\mu}$$

$$E(x) = \frac{6\tau^1 - 2}{\mu}$$

When $n = 3$

$$E(x) = \int_0^{\infty} \frac{2\tau^1 e^{-\mu x} (\mu x)^3}{6} x\mu dx - \int_0^{\infty} \frac{e^{-\mu x} (\mu x)^2}{2} \mu x dx$$

$$= \int_0^{\infty} \frac{\tau^1 e^{-\mu x} \mu^4 x^4}{6} dx - \int_0^{\infty} \frac{e^{-\mu x} \mu^3 x^3}{2} dx$$

$$= \frac{8\tau^1}{\mu} - \frac{3}{\mu}$$

$$E(x) = \frac{8\tau^1 - 3}{\mu}$$

When n = 4

$$\begin{aligned} E(x) &= \int_0^\infty x\mu [2\tau^1 P(4, x) - P(3, x)] dx \\ &= \int_0^\infty \frac{2\tau^1 e^{-\mu x} (\mu x)^4 \mu x}{24} dx - \int_0^\infty \frac{e^{-\mu x} (\mu x)^3 \mu x}{6} dx \\ &= \int_0^\infty \frac{\tau^1 e^{-\mu x} x^5 \mu^5}{24} dx - \int_0^\infty \frac{e^{-\mu x} \mu^4 x^4}{6} dx \\ &= \frac{10\tau^1}{\mu} - \frac{4}{\mu} \\ E(x) &= \frac{10\tau^1 - 4}{\mu} \end{aligned}$$

	n = 2	n = 3	n = 4	n = N
Mean	$\frac{6\tau^1 - 2}{\mu}$	$\frac{8\tau^1 - 3}{\mu}$	$\frac{10\tau^1 - 4}{\mu}$	$\frac{(2n+2)\tau^1 - n}{\mu}$

In general we write mean of the above probability distribution is

$$E(x) = \frac{(2n+2)\tau^1 - n}{\mu}$$

Conclusion

We proposed the mean for the Empirical Truncated Probability distribution for the assumed random variable. Mean of the above distribution is

$$E[x] = \frac{(2n+2)\tau^1 - n}{\mu}$$

Future Work

There is a lot of scope for future work for this topic. We try to formulate normalizing constant in terms of n. There is scope of formulating Variance, Standard deviation and Moment generating function of the above distribution.

References

- [1] Nirmala Kasturi, Distribution of Departures; International Journal Of Engineering and Advanced Technology, Volume 9, Issue 1S5, 2019.

- [2] MIT OPENCOURSEWARE, Lecture 10, Continuous Bayes rule, Derived Distributions.
- [3] William W. Hines, Douglas C. Montgomery, David M. Goldsman, Connie M. Borrer, Probability and Statistics in Engineering, Fourth Edition, Wiley Series.
- [4] Maria C.S. Lima, Gauss M. Cordeiro, Edwin M.M. Ortega & Abraão D.C. Nascimento, New extended normal regression model: simulations and applications, **Journal of Statistical Distributions and Applications**. Volume 6, Article number: 7 (2019), Springer.

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