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Mean To The Empirical Arrivals Distributions

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Abstract
This paper presents extended statistical properties for new empirical distributions by using general known familiar methods.

Keywords: Random variable; continuous probability distribution; arrival rate; density function; mean of the distribution.

Introduction
Statistics has the most interesting solutions for the problems in several fields due to its universality. Several new distributions have been developed by taking some subtle transformations on the existing distributions. This paper is continuous work to the previous work [1] which are briefed here.

Part 1[1]
The Random variable of interest is to "how likely there are successive arrivals in a particular interval". Instead of asking "how many arrivals take place in a particular time interval (Poisson)", we ask for "how likely the system to have successive arrivals in a particular interval of time". Since X is continuous, the PDF should be a function. We had made some inferences about this unknown function. This means the probability distribution that takes into account of measurements those we have surveyed for a considerable period of time. So the output of the inference problem is some distribution of X. We charted the histograms for different number of arrivals from the system from which we found the density curves by the use of regression techniques. [1]

In which case its probability density function is given by

$$f(x) = \begin{cases} \lambda\left[2\Im\left(P(n,x) - P(n-1,x)\right)\right] & when \;\; x \geq 0 \\ 0 \; otherwise \end{cases}$$

[1]
Where P (n, t) is the Poisson probability distribution of arriving n customers in [0, t]
P (n-1, t) is the Poisson probability distribution of arriving n-1 customers in [0, t]

And $\Im$ is the normalizing constant. [1]

Graph of the density function.

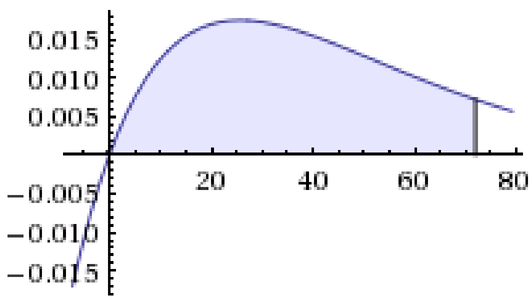

Fig. 3. Density function - 1

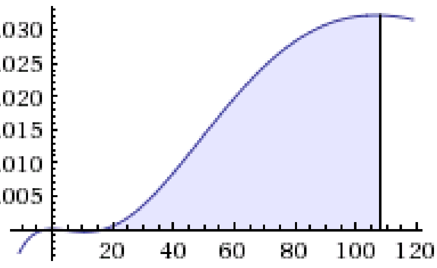

Fig. 4. Density function - 2

We can extend this probability distribution theory by calculating the interesting properties of all statistical distributions mean, variance and the standard deviation. These properties are studied due to their wide use in several fields like management, insurance, business and finance etc.

Mean of the Probability Distribution

Mean of the probability distribution is [3]

$$\text{Mean } E(x) = \int_0^\infty x f(x) dx \quad [3]$$

$$= \int_0^\infty x\lambda \left[2\tau P(n,x) - P(n-1,x)\right] dx$$

When n = 2

$$\text{Mean } E(x) = \int_0^\infty x\lambda\left[2\tau P(2,x) - P(1,x)\right] dx$$

$$= \int_0^\infty \lambda x 2\tau \left(\frac{e^{-\lambda x} (\lambda x)^2}{2} \right) dx - \int_0^\infty x\lambda e^{-\lambda x}, \lambda x \, dx$$

$$= \int_0^\infty \tau e^{-\lambda x} (\lambda x)^3 \, dx - \int_0^\infty e^{-\lambda x} (\lambda x)^2 \, dx$$

$$\frac{6\tau}{\lambda} - \frac{2}{\lambda}$$

$$E(x) = \frac{6\tau - 2}{\lambda}$$

When n = 3

$$\text{Mean } E(x) = \int_0^\infty \frac{2\tau e^{-\lambda\mu} (\lambda\mu)^3 \, x\lambda}{6} dx - \int_0^\infty \frac{e^{-\lambda\mu} (\lambda x)^2 \, \lambda x}{2} dx$$

$$= \int_0^\infty \frac{\tau e^{-\lambda\mu} \lambda^4 x^4}{6} dx - \int_0^\infty \frac{e^{-\lambda\mu} \lambda^3 x^3}{2} dx$$

$$= \frac{8\tau}{\lambda} - \frac{3}{\lambda}$$

$$E(x) = \frac{8\tau - 3}{\lambda}$$

When n = 4

Mean = $E(x) = \int_0^\infty x\lambda\left[2\tau P\left(4, x - P(3,x)\right)\right] dx$

$$= \int_0^\infty \frac{2\tau e^{-\lambda x}(\lambda x)^4 x\lambda}{24} dx - \int_0^\infty \frac{e^{-\lambda x}(\lambda x)^3 x\lambda}{6} dx$$

$$= \int_0^\infty \frac{\tau e^{-\lambda x}\lambda^5 x^5}{24} dx - \int_0^\infty \frac{e^{-\lambda x}\lambda^4 x^4}{6} dx$$

$$= \frac{10\tau}{\lambda} - \frac{4}{\lambda}$$

$$E(x) = \frac{10\tau - 4}{\lambda}$$

	n=2	n=3	n=4	n=N
Mean	$\frac{6\tau - 2}{\lambda}$	$\frac{8\tau - 3}{\lambda}$	$\frac{10\tau - 3}{\lambda}$	$\frac{(2n+2)\tau - n}{\lambda}$

In general we write the mean of the above probability distribution is

E(x) = $\frac{(2n+2)\tau - n}{\lambda}$

Conclusion

We proposed the mean for the Empirical Probability distributions for the assumed random variable.

$f(x) = \begin{cases} \lambda\left[2\Im\left(P(n,x) - P(n-1,x)\right)\right] & when \;\; x \geq 0 \\ 0 \; otherwise \end{cases}$ for the assumed random variable.

Mean of the above distribution is

E[x] = $\frac{(2n+2)\tau - n}{\lambda}$

Future Work

There is a lot of scope for future work for this topic. We try to formulate normalizing constant in terms of n. There is scope of formulating Variance, Standard deviation and Moment generating function of the above distribution.

References

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