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INVESTIGATION OF THE TESTS IN THE MODELING PROCESS IN UNIVARIATE TIME SERIES

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ABSTRACT

In this article, the tests used to test the suitability of univariate time series models are examined. For this purpose, considering the Box-Pierce Q(*m*), Ljung-Box Q(*m*), McLeod-Li Q(*m*), Monti's Q(*m*) and Li-McLeod Q(*m*) tests, the selected model structure and appropriate sample and the performance of the tests to ensure the suitability. In order to compare the tests with each other, Monte Carlo simulation method was used to obtain the first (I.) type error results of the tests and then the experimental power results and thus the comparison was made. According to this simulation study, the first experimental error and experimental power values were taken into consideration and the results of the tests were given.

Keywords: Time series, simulation, autocorrelation (ACF) and partial autocorrelation (PACF) coefficients, portmanteau test statistics

1. INTRODUCTION

Time series analysis is one of the most important disciplines used in statistics. Analyzing the data in accordance with time series, graphically expressing and modeling them is a task in itself. This issue, which is being used more and more widely every day, is especially preferred for the analysis and interpretation of time-based economy and business data.

When working in time series, model determination and the suitability of the model to the data are of great importance. Because a mis-determined model can have very different results. In this sense, the conformity of the model to the data must be tested immediately after the model determination stage. There are many tests for different model structures developed for this purpose.

The main theme of this study is to examine how the existing test methods will reveal the situation if different model structures are considered. The aim here; For univariate linear time series models, the tests in the modeling process are examined and compared in terms of their performance in determining whether the model is suitable for data. In this way, in addition to being a resource for researchers by examining the various test methods in the literature together in a specific study at the same time, it is to determine the comparable results of tests in different situations.

There are different models in time series: univariate and multivariable autoregressive moving average (ARMA) models, threshold type time series models, bilinear models, exponential autoregressive models, conditional autoregressive heteroscedasticity (ARCH) models. For these different models, the tests used to test whether the model is suitable for data are generally classified under portmanteau test statistics.

2. UNIVARIATE TIME SERIES AND MODELING

The time series is a collection of random variables, which are measurements of variables with data obtained in chronological order over time. A time series is generally shown in the format Z_t, $t = 1, \ldots, n$, with n sample sizes. Thus, *t-th* observed data over time is expressed by Z_t [4].

In univariate time series analysis, there is a dependence on past observation values since it is used to make future forecasts using past observation values. If so, there are some assumptions on which the univariate time series method is based. Accordingly, it is assumed that the elements of the time series present in a time series will remain the same in the future. Because of this assumption, it is provided to obtain the estimated values for the next period based on the previous period's agenda values. This method aims to estimate the future values of the series by separating the elements that make up the time series from each other and random elements. These methods are adapted to discrete time series consisting of observation values obtained at equal time intervals [4; 17].

Now, the properties and models of univariate general linear time series will be briefly introduced below.

2.1. Autocovariance, Autocorrelation and Partial Autocorrelation Coefficients

Three of the most common concepts in time series studies are autocovariance, autocorrelation and partial autocorrelation functions. They have properties that characterize the time series. Accordingly, autocovariance function,

Sample autocovariance function with the mean of a process ($\mu = E(Z_t)$: $\bar{Z} = \frac{1}{n}\sum_{t=1}^{n} Z_t$)

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=1}^{n-k}(Z_t - \bar{Z})(Z_{t-k} - \bar{Z})$$

(2.1)

It is also used in the following notation:

$$\hat{\gamma}_k = \frac{1}{n-k}\sum_{t=1}^{n-k}(Z_t - \bar{Z})(Z_{t+k} - \bar{Z})$$

(2.2)

It is expressed as.

The autocorrelation function returns the relationships between a time series and lagged series of that time series. For example; Autocorrelation between Z_t with Z_{t+k} refers to the relationship between the time series pairs $(Z_1, Z_{1+k}), (Z_2, Z_{2+k}), \ldots, (Z_{t-k}, Z_T)$. This is indicated by ρ_k and is the autocorrelation of the k. lagged. The autocorrelation coefficient values of all lags constitute the autocorrelation function. Sample autocorrelation function,

$$\hat{\rho}_k = \frac{\sum_{t=1}^{n-k}(Z_t - \bar{Z})(Z_{t+k} - \bar{Z})}{\sum_{t=1}^{n}(Z_t - \bar{Z})^2} = \frac{\hat{\gamma}_k}{\gamma_0} \qquad k = 1,2,\ldots$$

(2.4)
shown in the form. Standard errors of sample autocorrelation

$$S_{\hat{\rho}_k} \cong \sqrt{\frac{1}{n}(1 + 2\sum_{i=1}^{n}\hat{\rho}_i^2)}$$

(2.5)
or briefly,

$$S_{\hat{\rho}_k} \cong \sqrt{\frac{1}{n}}$$

(2.6)
As is known, the correlation coefficient (ρ) takes values between -1 and +1. Since the autocorrelation coefficient expresses the relationship between time series, it takes values between -1 and +1 [20].

The partial correlation coefficient, as known from the regression, gives the relationship between the two variables while the other variables are constant. The partial autocorrelation coefficient is, again from a similar perspective, the relationship between the time series and the time series of the respective lag, ignoring the effects of other lagged series. Sample partial autocorrelation function;

$$\hat{\phi}_{k+1,k+1} = \frac{\hat{\rho}_{k+1} - \sum_{j=1}^{k}\hat{\phi}_{kj}\hat{\rho}_{k+1-j}}{1 - \sum_{j=1}^{k}\hat{\phi}_{kj}\hat{\rho}_j}$$

(2.7)

$$\hat{\phi}_{k+1,j} = \hat{\phi}_{kj} - (\hat{\phi}_{k+1,k+1})(\hat{\phi}_{k,k+1-j}) \qquad j=1,\ldots,k$$

(2.8)

$$S_{\hat{\phi}_{kk}} \cong \frac{1}{\sqrt{n}}$$

(2.9)
as shown. Partial autocorrelation coefficient values of all lagged constitute the partial autocorrelation function [20].

2.2. Univariate General Stationary Models

As is known, in order for any time series to be stationary, the mean, variance, covariance and higher order moments must be constant over time. In univariate stationary time series, autoregressive (AR), moving average (MA) and autoregressive-moving average (ARMA) models are called linear stationary models. If the model is not stationary, the series must be stationary [4]. The models are summarized below.

Autoregressive (AR) models: If any time series can be expressed in terms of lags, this model is called the autoregressive (AR) model. AR models are named according to the number of past periods they contain. If there is only one observational AR model from the previous period, it is called "first order", and if *p* is related to past observation value, it is called "*p-th* order AR model.

The general expression of the AR(*p*) model is as follows;

$$z_t = \phi_1 z_{t-1} + ... + \phi_p z_{t-p} + A_t$$

(2.10)

or

$$\phi_p(B) z_t = A_t$$

(2.11)

Here, the values Z_t, Z_{t-1},..., Z_{t-p} are obtained by taking the difference of μ from each observation value ($Z_t = Y_t - \mu$ to represent the original sequence Y_t). ϕ_1, ϕ_2 ,..., ϕ_p are unknown but estimated parameters of the model. *p* is the degree of the model and A_t is the white noise process with a mean of zero and a variance of σ^2. It can be written as follows,

$$BZ_t = Z_{t-1}, \quad B^2Z_t = Z_{t-2}, \quad ..., \quad B^pZ_t = Z_{t-p}$$

(2.12)

including

$$Z_t = (\phi_1 B + \phi_2 B^2 + ... + \phi_p B^p) Z_t + A_t$$

(2.13)

This way of writing helps to determine whether the models meet the stationary condition. To be stationary, the roots of $\phi(B) = 0$ must lie outside of the unit circle. The AR processes are useful in describing situations in which the present value of a time series depends on its preceding values plus a random shock [20] [9].

Moving average (MA) model: The models in which any time series is expressed as a linear combination of the error term of the same period and the error terms of a certain number of past periods are the moving average models. Unlike AR model, moving average model is obtained by using current and past error terms instead of past observation values. Moving average models are also determined by the number of past error terms they contain. If MA model is connected to only one error term from the previous period, it is called "first order" and if MA model is connected to past q error terms, it is called "*q*-th order" MA model.

The general expression of the MA(*q*) model can be written as follows.

$$Z_t = A_t - \theta_1 A_{t-1} - \theta_2 A_{t-2} - ... - \theta_q A_{t-q}$$

(2.14)

Here, $\theta_1, \theta_2, ..., \theta_q$ are the parameters of the model.

$Z_t = (1 - B\theta_1 - ... - \theta_q B^q)A_t$
(2.15)
is written.

Here, θ_1, θ_2 ,..., θ_q are the estimated parameters of the model. *q* represents the degree of MA model. MA models always stationary. This moving average process is invertible if the roots of $\theta(B)=0$ lie outside of the unit circle. MA(*q*) is given by,
$Z_t = (1 - B\theta_1 - ... - \theta_q B^q)A_t$
(2.16)

Autoregressive-moving average(ARMA) models: The general mixed model of AR and MA models of stationary time series is the ARMA model. That is, ARMA(*p,q*) is combination of AR(*p*) and MA(*q*). In these models, there is a linear combination of the terms of observation, a certain number of observations, and errors before any period of time. The degree of the model is determined by past observation values (*p*) and past error values (*q*). The ARMA(*p,q*) model can generally be written as follows:
$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + ... + \phi_p Z_{t-p} + A_t - \theta_1 A_{t-1} - ... - \theta_q A_{t-q}$
(2.17a)
or
$Z_t - \phi_1 Z_{t-1} - \phi_2 Z_{t-2} - ... - \phi_p Z_{t-p} = A_t - \theta_1 A_{t-1} - ... - \theta_q A_{t-q}$
(2.18b)
(2.17a) and (2.18b) the meanings of the notations in the equations are as stated above.
Here $\phi_1, \phi_2, ..., \phi_p$ and $\theta_1, \theta_2, ..., \theta_q$ are the parameters. ARMA(*p,q*) is given by,

$$\phi(B)Z_t = \theta(B)A_t \quad (2.19)$$

Where ϕ(B)and θ(B) are polynomials of *p* and *q* degrees, respectively. These;
$\phi(B)=1-\phi_1 B-\phi_2 B^2-...-\phi_p B^p$
(2.20)
$\theta(B)=1-\theta_1 B-\theta_2 B^2-...-\theta_q B^q$
(2.21)

They are expressed as. ARMA(*p*, *q*) model; for stationarity, the roots of $\phi(B) = 0$ must lie outside of the unit circle and for invertibility, the roots of $\theta(B) = 0$ must lie outside of the unit circle. The availability conditions of stationarity and invertibility should be considered together [20].

2.3. Univariate Non-Stationary Linear Models

Time series which is encounter in real life are often not stable. The stability of these series is disrupted by trend, seasonal and conjuncture fluctuations and random events. For stationarity, these factors must be identified and eliminated beforehand, in short, a non-stationary time series should be converted to stationary. This can be done by examining the ACF (autocorrelation function) and PACF (partial autocorrelation function) of the sequence [4] or by applying the unit root test and taking the appropriate degree differences of the series [8].

The degree of autoregressive parameter is p, the degree of the moving average parameter is q and d is the number of differences received, this model is called the autoregressive integrated moving average model and is called ARIMA (p,d,q). General ARIMA(p,d,q) model,
$\phi_p(B)$ and $\theta_q(B)$ as defined above:

$$\phi_p(B)(1-B)^d Z_t = \theta_0 + \theta_q(B) + A_t \quad (2.22)$$

There is no significant contribution here that is $\phi_p(B)$, $\theta_q(B)$. θ_0 parameter d = 0 and d> 0 have very different roles. If d = 0, the process is stationarity, and θ_0 is associated with the mean of the process as $\theta_0 = \mu(1-\phi_1-...-\phi_p)$. However, if $d \geq 1$ than θ_0 is called the deterministic trend term. ARIMA(p,d,q) expression

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + ... + \phi_p x_{t-p} + A_t + \theta_1 A_{t-1} + ... + \theta_q A_{t-q} \quad (2.23)$$

including;

$x_t = \Delta^d z_t$:differenced series; Δ : difference operator; d : degree of difference

$$\Delta z_t = x_t = z_t - z_{t-1} \quad (2.24)$$

is defined as [20];

2.4. Modeling Process

The first thing to do in the modeling process in a time series is to draw the graphs of the sequences. If the graph tends to increase or decrease continuously, it can be said that the sequence is not stationary. But this can lead to some misconceptions and unhealthy decisions. Because at first glance, stationarity process may show some changes over time. One way to detect stationarity in time series is to look at the autocorrelation (ACF) and partial autocorrelation (PACF) graphs of the series. Only stationary process lead to statistically significant ACF and PACF. Non-stationary series must therefore be converted to stationary arrays by applying appropriate difference-taking methods.

Which model the data refers to can be said by looking at the graphical behavior of ACF and PACF. Accordingly, oscillating reduction of ACF or PACF indicates AR or MA processes. If both (ACF and PACF) have oscillating drops, this indicates a mixed ARMA process. Further, by the number of autocorrelations and / or partial autocorrelations of non-zero significance, the value of the coefficient p for the AR scheme is determined from the PACF structure and the value of the coefficient q for the MA scheme is determined from the ACF structure. If the roughly appropriate model is of MA(q) type, the degree of this model is determined by using autocorrelation coefficients. For the ARMA(p,q) model, autocorrelation and partial autocorrelation coefficients are used to determine p and q degrees. In this case, as a temporary model; One suitable model AR(p), MA(q) and ARMA(p,q) can be detected [8] [10] [1].

If the autocorrelation coefficients do not tend to exponentially approach zero, then there is a non-stationary series. In other words, if the coefficients decrease slowly exponentially, there is a non-stationary state. This series needs to be stationarity for analysis. [4].

In univariate time series, the parameters are estimated after determining the degree of the model at the modeling stage. Parameter estimation can generally be considered in three different headings; moments method, maximum likelihood method and the least known squares method.

After completing the parameter estimations of the temporary model from the modeling stages, the suitability of this model to the data is tested by using the residual terms. So at this stage, it is examined whether the obtained model is suitable or not. The examination of the suitability of the model is an examination of the whole model [5] [4] [20].

3. TESTING THE COMPATIBILITY OF THE MODEL IN UNIVARIATE VARIABLE TIME SERIES

One of the most important stages of model building is to be able to identify it correctly. In particular, it is concerned whether the residues of the predicted model are white noise or not. That is to say, which time series model fits at hand. The first choice for this is to look at the ACF and PACF graphs other than the graphs of the sequences themselves. The graph shows the delays of the ACF and PACF. However, these graphs also contribute to providing information about stasis structures. The second preference is to establish a test statistic to test the null (H_0) hypothesis where the residues m are equal to zero at *m* lag. In particular, it may be not only stationary, but also non-stationary structures. These tests can be applied separately to check the non-stationary on average and non-stationary on variance [2].

The test statistic used in the second approach is expressed as the Portmanteau statistic. It is worth mentioning some Portmanteau statistics here. In addition to the original Box-Pierce (1970) and modified Ljung-Box (1986) Portmanteau statistics, there are also new Portmanteau statistics available. In these tests, the null hypothesis is tested whether autocorrelation or partial autocorrelation coefficients are significant [3].

3.1. Box-Pierce Test Statistics

This test statistic was proposed by Box-Pierce (1970) in order to test the suitability of the data to the model in the time series modeling process. Portmanteau test statistics use autocorrelation of residuals and partial autocorrelation coefficients of residuals. Autocorrelation coefficient of residuals;

$$r_k = \sum_{t=k+1}^{n} a_t a_{t-k} / \sum_{t=1}^{n} a_t^2 \,, \qquad k=1,...,m \tag{3.1}$$

is expressed as. If the model is sufficient and n$\geq$m, it is the expectation as $r_1 \cong r_2 \cong .... \cong r_m \cong o$.

Therefore, the adequacy of the model is based on the importance of r_k [2].

The Box-Pierce test statistic described here is a test statistic based on the use of the autocorrelation coefficient of residuals.

3.2. Ljung-Box Test Statistics

Box and Pierce (19709 proposed a test statistic based on the autocorrelation coefficient of residuals as mentioned above. The Q_{BP} statistic was shown to distributed asymptotic chi-square with the degree of freedom of *m-p-q* if the model is appropriate and *m*>0. In relation to this statistic, discussions have been put forward by researchers working on it in the following years.

Thereupon, some scientists proposed to make changes in this test statistic and made studies in this direction.
Ljung and Box (1978) and Prothero and Wallis (1976) proposed the use of the modified *Q* statistic and derived the following test statistic,

$$Q_{LB} = n(n+2)\sum_{k=1}^{m} r_k^2 /(n-k)$$

(3.2)

Q_{LB}, has a limited sample distribution that χ^2_{m-p-q} is much closer than. This form of Q_{BP} statistic is preferred by some practitioners, which is referred to as Ljung-Box or Ljung-Box-Pierce statistics. The reason for modification is from $Var(r_k) \cong (n-k)/\{n(n+2)\}$. That is, Q_{LB} is obtained in particular by correcting each of the autocorrelation coefficients of the residuals (r_k) in the Q_{BP} with its asymptotic variance. Davies, Triggs and Newbold (1977) showed that the variance of Q_{LB} was significantly greater than the *m-p-q* degree of freedom and chi-square distribution, which was 2(*m-p-q*) [13].

3.3. Li-McLeod Test Statistics

This test statistic is another modification of the Box-Pierce *Q* statistic. Li and McLeod (1981), while considering changes to the known Box-Pierce *Q(m)* test statistic, propose to examine the expected value of this test statistic. Accordingly, Li and McLeod (1981), with the idea that the change in the expected value will create a significant deviation in test statistics,

$$Q_{LM} = Q_{BP} + \frac{m(m+1)}{2n}$$

(3.3)

proposed change. When Box-Pierce Q (m) Portmanteau test statistic is replaced,

$$Q_{LM} = \frac{m(m+1)}{2n} + n\sum_{k=1}^{m} r_k^2$$

It has proved to be easier than Box-Pierce *Q* Portmanteau statistics in terms of application and programming [2].

3.4. McLeod-Li Test Statistics

This test method was proposed by McLeod and Li (1983). The general approach here is to use squared residual autocorrelations to control the modeling process based on the assumption of the linear ARMA model. m lagged squared residual autocorrelations,

$$r_{aa}(m) = \frac{\sum_{t=m+1}^{n}(a_t^2 - \hat{\sigma}^2)(a_{t-m}^2 - \hat{\sigma}^2)}{\sum_{t=m+1}^{n}(a_t^2 - \hat{\sigma}^2)^2}$$

(3.4)

is expressed as. Here, $\hat{\sigma} = \sum a^2 / n$. In this section, it is worth mentioning the distribution of squared residual autocorrelations. Q_{ML} statistics

$$Q_{ML} = n(n+2)\sum_{k=1}^{m} \frac{r_{aa}^2(m)}{n-m}$$
(3.5)

4. SIMULATION STUDY

4.1. Simulation Plan

Box-Pierce *Q(m)*, Ljung-Box *Q(m)*, Monti's *Q(m)*, Li-McLeod *Q(m)* tests are compared using Monte Carlo simulation method, considering different situations. The main purpose is to test the null hypothesis whether the model is suitable for the data. Assume that there are different situations in the data production process. Accordingly, data is produced under the assumption that the appropriate model is the AR(1) model. Considering these situations, a simulation study was performed and experimental first type error and experimental power results were obtained for each test. 10000 experiments are conducted in the simulation study.

4.2. Experimental Results

Experimental first (I.) type error (*α*) results: In this section, data are generated in 10000 trials using the Monte Carlo simulation method from the AR(1) model and compared for the five test methods that indicate the suitability of each model over these data. The way of calculating experimental first-type error (*α*) results: While the H_0 hypothesis is known to be correct, the mean of the autocorrelation coefficient is zero(0), the variance is one(1) and the standard Normal distribution is different for each test in order to find the rate of rejection that is actually correct for each test. For each combination of varying lag and parameter values, the sample selection is repeated 10000 times and the number of rejecting the H_0 hypothesis is obtained by dividing this number by 10000. The nominal first type error is taken as α=0.05. First type error (*α*) results obtained by this method are given in the tables below.

Model AR (1):
In this section, different autoregressive parameters are considered and using the Monte Carlo simulation method for four AR(1) models, Box-Pierce *Q(m)*, Ljung-Box *Q(m)*, McLeod-Li *Q (m)*, Monti's *Q(m)* and Li-McLeod *Q(m)* tests, the sample volume was taken as *n*=100 and experimental first type error (*α*) results were obtained. Here are the models used in data generation:
Model-1: $z_t = 0.20 z_{t-1} + a_t$
Model-2: $z_t = 0.50 z_{t-1} + a_t$
Model-3: $z_t = 0.70 z_{t-1} + a_t$
Model-4: $z_t = 0.99 z_{t-1} + a_t$
Experimental α results obtained for each test using these models are given in Table 4.1 for sample volume *n*=100.

Table 4.1. For *n*=100, when the model is AR(1) in 10000 trials Box-Pierce *Q*(*m*), Ljung-Box *Q*(*m*), McLeod-Li *Q*(*m*), Monti's *Q*(*m*) and Li-McLeod *Q*(*m*) tests experimental first type error (*α*) results

	Autoregressive Parameter	lag			
	ϕ	**m=2**	**m=3**	**m=5**	**m=10**
Box-Pierce Q(m)	**0.20**	0.050	0.050	0.050	0.050
	0.50	0.056	0.051	0.049	0.049
	0.70	0.078	0.069	0.065	0.051
	0.99	0.132	0.098	0.075	0.064
Ljung-Box Q(m)	**0.20**	0.051	0.048	0.048	0.050
	0.50	0.052	0.050	0.051	0.054
	0.70	0.080	0.055	0.054	0.058
	0.99	0.129	0.094	0.072	0.068
McLeod-Li Q(m)	**0.20**	0.045	0.044	0.046	0.049
	0.50	0.050	0.049	0.049	0.055
	0.70	0.075	0.050	0.052	0.056
	0.99	0.129	0.082	0.066	0.065
Monti's Q(m)	**0.20**	0.050	0.050	0.049	0.051
	0.50	0.051	0.050	0.050	0.052
	0.70	0.079	0.057	0.055	0.054
	0.99	0.119	0.078	0.079	0.065
Li-McLeod Q(m)	**0.20**	0.049	0.049	0.048	0.048
	0.50	0.051	0.048	0.047	0.046
	0.70	0.082	0.052	0.051	0.050
	0.99	0.120	0.089	0.081	0.063

According to Table 4.1; When the sample volume is n 100, it is observed that the experimental results of the tests are compared for the AR(1) model. According to this; When the autoregressive parameter value is $\phi \leq 0.70$, the model is considered to be appropriate for all lag counts at the significance level determined. This is provided for all tests.

When the autoregressive parameter is ϕ=0.99, it is clear that the experimental results obtained in cases where the number of delays are small have deviated considerably from the nominal level.

Therefore, the ability of tests to approach acceptable levels depends on the increasing number of delays.

Experimental power (1-β) results: In this section, firstly, Monte Carlo simulation method is used to generate data for 10000 trials, which is assumed to be appropriate model, then power results are obtained for five different test methods for different nine(9) models. The way of calculating the experimental power (1-β) results is: when the hypothesis is known to be incorrect, in order to find the rate of rejection of the hypothesis that is actually incorrect for each test, after generating the data for the appropriate model, the power is compared to the different models discussed in order to compare the tests in terms of power. results are obtained. Here, for comparison, the power results of the tests are obtained for different sample volumes and different lag numbers in order to see the effect of different situations.

Again, from a similar point of view, AR(1) was used as the appropriate model and power results were obtained. Therefore, the performance of the tests in different situations was investigated.

Suitable model AR(1):

In this section, when the appropriate model is the AR(1) model (ie, obtained from the data generation model), power results are obtained in 10000 trials with Monte Carlo Simulation method for the following models. Here the nominal first type error is taken as $\alpha = 0.05$.

Model-1: $z_t = a_t - 0.40a_{t-1}$
Model-2: $z_t = a_t - 0.80a_{t-1}$
Model-3: $z_t = a_t - 0.60a_{t-1} - 0.30a_{t-2}$
Model-4: $z_t = 0.10z_{t-1} + 0.30z_{t-2} + a_t$
Model-5: $z_t = 0.70z_{t-1} + a_t - 0.40a_{t-1}$
Model-6: $z_t = 0.70z_{t-1} + a_t - 0.90a_{t-1}$
Model-7: $z_t = 0.40z_{t-1} + a_t - 0.60a_{t-1} - 0.30a_{t-2}$
Model-8: $z_t = 0.70z_{t-1} + 0.20z_{t-2} - 0.50a_{t-1} + a_t$
Model-9: $z_t = 0.90z_{t-1} + 0.40z_{t-2} - 0.70a_{t-1} - 0.30a_{t-2} + a_t$

Table 4.2. When the appropriate model is AR(1) in 10000 trials for *n* 100, Box-Pierce *Q*(*m*), Ljung-Box *Q*(*m*), McLeod-Li *Q*(*m*), Monti's *Q*(*m*) and Li -McLeod *Q*(*m*) tests experimental power results (α=0.05, *m*=10)

Model	***Φ***	***Φ***	***θ***	***θ***	**Box-Pierce *Q(m)***	**Ljung-Box *Q(m)***	**McLeod-Li *Q(m)***	**Monti's *Q(m)***	**Li-McLeod *Q(m)***
1	-	-	**0.40**	-	0.39	0.40	0.36	0.44	0.41
2	-	-	**0.80**	-	0.76	0.82	0.74	0.84	0.81

3	**-**	**-**	**0.60**	**0.30**	0.79	0.83	0.77	0.86	0.82
4	**0.10**	**0.30**	**-**	**-**	0.42	0.45	0.43	0.42	0.44
5	**0.70**	**-**	**0.40**	**-**	0.56	0.55	0.49	0.56	0.54
6	**0.70**	**-**	**0.90**	**-**	0.94	0.96	0.90	0.98	0.95
7	**0.40**	**-**	**0.60**	**0.30**	0.82	0.86	0.83	0.91	0.85
8	**0.70**	**0.20**	**0.50**	**-**	0.51	0.50	0.46	0.53	0.52
9	**0.90**	**0.40**	**0.70**	**0.30**	0.71	0.69	0.65	0.75	0.73

According to Table 4.2; When the sample volume is *n*=100, the experimental power results of five test methods for nine predetermined time series models are given as a result of Monte Carlo simulation study conducted with 10000 experiments under the assumption that the appropriate model is the AR(1) model. Almost all of the results obtained here seem to be meaningful when considering the power values, but it is observed that the tests show a variable structure for these different situations. Although these different situations arise from the model structures, the selected sample volume and the number of lag are other factors affecting the results obtained.

According to experimental power results obtained for Model-1, not all tests are very strong compared to other models. Looking at Model-2, it is observed that the power increases with the increase of the moving average parameter. According to the experimental power results, Monti's Q(*m*) test is more powerful. The same applies to Model-3. However, when the Model-4 is examined, it is observed that the value of the experimental power results decreased.

In Model-5 and Model-6, results were obtained for different parameters of ARMA(1,1) time series model. Although Model-5 is not very strong, it is observed that the value of experimental power increases when the parameter value increases in Model-6. Similarly, while a decrease in the value of experimental power obtained in Model-8 is observed, it can be said that the results obtained in Model-7 and Model-9 are significant.

5. CONCLUSION

In this article, the tests used to test the suitability of univariate time series models are examined. Among the tests used in the literature, Box-Pierce $Q(m)$, Ljung-Box $Q(m)$, McLeod-Li $Q(m)$, Monti's $Q(m)$ and Li-McLeod $Q(m)$ tests were taken into consideration. Simulation studies were conducted to evaluate the performance of these tests in terms of ensuring compliance with the selected model structures. In order to compare the tests with each other, Monte Carlo simulation method was used to obtain the first (I.) type error (α) results of the tests and then the results of experimental power $(1\text{-}\beta)$ and the comparison was aimed.

In order to obtain the experimental first (I.) type error (α) results of the tests; primarily, the AR(1) model is taken as an example. If the different situations of this model are taken into consideration, the experimental results of the five test methods are given together so that the tests can be compared with each other more easily. In order to evaluate the results of experimental first (I.) type error (α) obtained for these tests, test methods, related parameters in terms of models and delay numbers were examined. Accordingly, it has been demonstrated how the tests differ according to each other in terms of differences in parameters or parameters, number of delays and sample volume. In this evaluation, (I.) type error (α) was taken as 0.05. Further; tests were compared in terms of their strength. For this purpose, experimental power results were obtained for various time series models under the assumption of the appropriate model.

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