IEEESEM

CUMULATIVE DISTRIBUTION ON DEPARTURES

Abstract: This paper proposes the new cumulative distributive function for density function

$f(x)=\frac{e^{-\mu x}(\mu x)^{N-n}\mu}{(N-n)!}$ for the chosen random variable "departing that (N-n) no. of persons in the system.".

Key Words: Random variable, Continuous probability distribution, Arrival rate, Density function, Cumulative Distribution Function.

Introduction This paper is continuous work to the previous one [2] which is briefed here. The Random variable of interest is to "departing that (N-n) no. of persons in the system." Instead of asking "how many arrivals take place in a particular time interval (Poisson)", we ask for "how likely the system departing that (N-n) no. of persons." Since X is continuous, the PDF should be a function. We had made some inferences about this unknown function. This means the probability distribution that takes into account of measurements those we have surveyed for a considerable period of time. So the output of the inference problem is the distributions of X. We charted the histograms for departing that (N-n) no. of persons in a particular time interval from which we found the density curves.

In which case its probability density function is given by

in which case its probability density function is given by

$$f(x)=\frac{e^{-\mu x}(\mu x)^{N-n}\mu}{(N-n)!}$$

Where N is the restricted number of arrivals to the system.

μ is the departure rate.

n is no. of persons remained in the system after taking (N-n) persons their service.

Graph of density function:

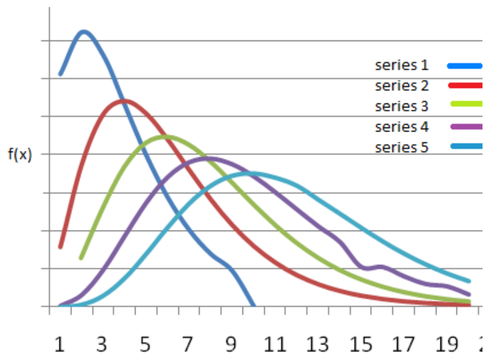

Figure - 1

Series 1 represents the probability density function graph when n = 0.

Series 2 represents the probability density function graph when n = 1.

Series 3 represents the probability density function graph when n = 2.

Series 4 represents the probability density function graph when n = 3.

Series 5 represents the probability density function graph when n = 4.

X- axis represents Time, Y- represents f(x).

Cumulative Distribution function:

It is the probability that random variable takes values less than or equal to x

$$f_X(x) = (X \leq x) = \int_{-\infty}^{x} f_X(t)\,dt$$

The density function of assumed random variable is

$$f(x) = \begin{cases} \dfrac{\mu e^{-\mu x}(\mu x)^{N-n}}{(N-n)!}\,dx\,When\ x \geq 0 \text{ and n=0 n = } 0,1,2...\text{N} \\ 0\,otherwise \end{cases}$$

For n = N – 0

i.e. n=N.

(N-n) =N-(N-0).

$$F_X(x) = \left\{1 - \int_x^\infty \mu e^{-\mu x} dx\right.$$

$$= 1 - \left[\frac{\mu e^{-\mu x}}{\mu}\right]_x^\infty$$

$$F_X(x) = 1 - e^{-\mu x}$$

This gives up to certain time interval [o, x] the probability of departing zero customer from the system.

For n = N-1

(N-n) = N-(N-1)

$$f_X(x) = P(X \le x) = 1 - \int_x^\infty \frac{\mu e^{-\mu x}.\mu x}{1!} dx$$

$$1 - \mu^2 \int_x^\infty x e^{-\mu x}.dx$$

$$= 1 - \mu^2 \left[\frac{-xe^{-\mu x}}{\mu} - \int \frac{e^{-\mu x}}{-\mu} dx\right]_x^\infty$$

$$= 1 - \mu^2 \left[\frac{-xe^{-\mu x}}{\mu} - \frac{e^{-\mu x}}{\mu^2}\right]_x^\infty$$

$$= 1 - \mu^2 \left[\frac{\mu e^{-\mu x}}{\mu} + \frac{e^{-\mu x}}{\mu^2}\right]_x^\infty$$

$$1 - \mu x e^{-\mu x} - e^{-\mu x}$$

$$F_X(x) = \begin{cases} 1 - \sum_{k=0}^{N-(N-1)} \dfrac{e^{-\mu x}.\left(e^{-\mu x}\right)^k}{k!} \end{cases}$$

$$F_X(x) = \begin{cases} 1 - \sum_{k=0}^{N-(N-1)} \dfrac{e^{-\mu x}.\left(e^{-\mu x}\right)^k}{k!} & when \ x \geq 0 \\ 0 \ \ otherwise \end{cases}$$

This gives up to certain time interval [0, x] the probability of departing one customer from the system.

For n = N-2

(N-n) =N-(N-2)

$$f_X(x) = P\left(X \leq x\right) = 1 - \int_x^{\infty} \frac{\mu e^{-\mu x}(\mu x)^2}{2!} dx$$

$$1 - \mu^3 \int_x^{\infty} \frac{x^2 e^{-\mu x}}{2!} .dx$$

$$1 - \frac{(\mu x)^2 e^{-\mu x}}{2!} - \mu x e^{-\mu x} - e^{-\mu x}$$

$$F_X(x) = \begin{cases} 1 - \sum_{k=0}^{N-(N-2)} \dfrac{e^{-\mu x}\left(\mu x\right)^k}{k!} \end{cases}$$

$$F_X(x) = \begin{cases} 1 - \sum_{k=0}^{2} \dfrac{e^{-\mu x}.\left(e^{-\mu x}\right)^k}{k!} & when \ x \geq 0 \\ 0 \ \ otherwise \end{cases}$$

This gives up to certain time interval [0, x] the probability of departing two customer from the system.

For n = N-3

(N-n) =N-(N-3)

$$f_X(x) = P\left(X \le x\right) = 1 - \int_x^\infty \frac{\mu e^{-\mu x}(\mu x)^3}{3!} dx$$

$$= 1 - \mu^4 \int_x^\infty \frac{x^3 e^{-\mu x}}{3!} .dx$$

$$= 1 - \frac{(\mu x)^3 e^{-\mu x}}{3!} - \frac{(\mu x)^2 e^{-\mu x}}{2!} - \mu x e^{-\mu x} - e^{-\mu x}$$

$$F_X(x) = \begin{cases} 1 - \sum_{k=0}^{N-(N-3)} \frac{e^{-\mu x}\left(\mu x\right)^k}{k!} \end{cases}$$

$$F_X(x) = \begin{cases} 1 - \sum_{k=0}^{3} \frac{e^{-\mu x}.\left(\mu x\right)^k}{k!} \; when \;\; x \ge 0 \\ 0 \;\; otherwise \end{cases}$$

This gives up to certain time interval [0, x] the probability of departing three customer from the system.

For n = N-4

(N-n) = N-(N-4)

$$f_X(x) = P\left(X \le x\right) = 1 - \int_x^\infty \frac{\mu e^{-\mu x}(\mu x)^4}{4!} dx$$

$$= 1 - \mu^5 \int_x^\infty \frac{x^4 e^{-\mu x}}{4!} dx$$

$$= 1 - \frac{(\mu x)^4 e^{-\mu x}}{4!} - \frac{(\mu x)^3 e^{-\mu x}}{3!} - \frac{(\mu x)^2 e^{-\mu x}}{2!} - \mu x e^{-\mu x} - e^{-\mu x}$$

$$F_X(x) = \begin{cases} 1 - \sum_{k=0}^{N-(N-4)} \frac{e^{-\mu x}\left(\mu x\right)^k}{k!} \end{cases}$$

$$F_X(x) = \begin{cases} 1 - \sum_{k=0}^{4} \frac{e^{-\mu x}.(\mu x)^k}{k!} & when\ x \geq 0 \\ 0 & otherwise \end{cases}$$

This gives up to certain time interval [0, x] the probability of departing four customer from the system.

In general the corresponding Cumulative distributive function of above density function is defined as $F_X(x) = \begin{cases} 1 - \sum_{k=0}^{N} \frac{e^{-\mu x}.(\mu x)^k}{k!} & when\ x \geq 0 \\ 0 & otherwise \end{cases}$

Graph of Cumulative distributive function:

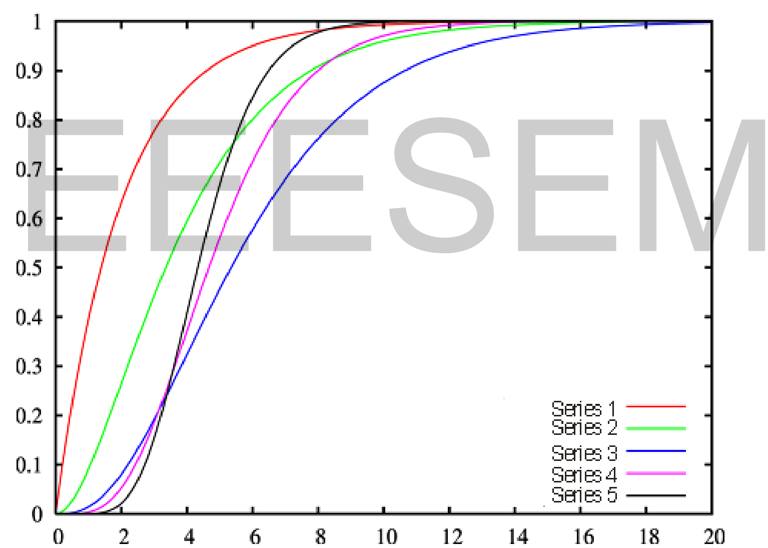

Figure 2

Series 1 represents the Cumulative distributive function graph when N = 0.

Series 2 represents the Cumulative distributive function graph when N = 1.

Series 3 represents the Cumulative distributive function graph when N = 2.

Series 4 represents the Cumulative distributive function graph when N = 3.

Series 5 represents the Cumulative distributive function graph when N = 4.

CONCLUSION

This paper proposed the cumulative distribution function for density function

$f(x)=\frac{e^{-\mu x}(\mu x)^{N-n}\mu}{(N-n)!}$ for the chosen random variable of "departing that (N-n) no. of persons in the system.". And the corresponding cumulative distribution function is

$$F(x)=\begin{cases}1-\sum_{k=0!}^{N-n}\frac{e^{-\mu x}(\mu x)^{k}}{k!} & When\ x\geq 0\\ 0\ otherwise\end{cases}$$

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