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A CONTINUOUS DISTRIBUTION ON DEPARTURES

ABSTRACT Probability is useful among other things since we use it to make sense of the world around us. We use it to make inferences about the things that we do not see directly. And this is done by using probability distributions. We have already seen some of the distributions. Several distributions have been developed by some transformations on the existing distributions. This paper proposes a new truncated continuous probability density function for the eclectic random variables.

Key Words: Random variable, continuous probability distribution, Departure rate, density function, histograms.

Introduction: Instead of asking "In fixed time, how many departures will take place?" We ask how likely the interval to have 'N-n' fixed no. of departures out of N arrivals. That is only N-n no. of departures from the system in particular interval. Since X is continuous, the PDF should be a function. We have to make inferences about this unknown function. This means the probability distribution that takes into account of measurements those we have surveyed for a considerable period of time. So the output of the inference problem is to come up with the distributions of X. We plot the histograms for different number of departures and smoothing techniques of histograms we found the density curves. Our random variable is departing that (N-n) no. of persons in the system. We can say this is continuous random variable, if there exists a probability density function which satisfies the following properties.

(i) $f(x) \geq 0 \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

We propose

$$f(x) = \frac{e^{-\mu x}\left(\mu x\right)^{N-n}\mu}{\left(N-n\right)!}$$

a truncated probability density function. We restrict the (domain) arrivals no. to N. After finishing service of N persons, again we allow another set of persons in to the queuing system.

Where μ is the departure rate.

N is the no. of persons remained in the system after taking (N-n) persons their service. (N-n) is the no. of persons who received service f(x) is continuous on [a, b] and also f(x) is integrable on $[a,b] \subseteq R$.

For n = N-N = 0:

Zero departures happen from the system in the following ways.

If no person enters the system $\Rightarrow$ '0'departure happen from the system

In particular interval

If N persons enters in to the system

and no person receives the $\Rightarrow$ '0' departures happen from the system.

service in particular system.

. . . .

. . . .

And so on.

$$f(x) = \frac{e^{-\mu x}(\mu x)^{N-n}\mu}{(N-n)!}$$

Here the no. of persons remained in the system is n=N $\therefore N - N = 0$

$$f(x) = \frac{e^{-\lambda x}(\mu x)^{N-N}\mu}{(N-N)!}$$

$$f\left(x\right)=\frac{e^{-\mu x}\left(\mu x\right)^{0}\mu}{0!}$$

(1) $f\left(x\right)=e^{-\mu x}.\mu$

The densities are used to calculate probabilities and probabilities must be non negative. Therefore, the density should also be non negative.

For chosen random variable, this f(x) is to be probability density function if

$f(x)\geq 0\forall x$

$e^{-\mu x}.\mu \geq 0\forall x$ Since exponential gives all positive values and μ is service rate which is positive, time $x\geq 0$ and n = 0, 1, 2….N which are positive.

(2) $Now\int_{-\infty}^{\infty} f(x)dx=\int_{-\infty}^{\infty} e^{-\mu x}\mu dx=\mu\left[\frac{e^{-\mu x}}{-\mu}\right]_{0}^{\infty}$

$$\mu\left[\frac{e^{-\mu x}}{-\mu}\right]_{0}^{\infty}=-\left[e^{-\infty}-e^{0}\right]=1$$

$\therefore\ f(x)=e^{-\mu x}$ is the probability density function for the assumed random variable

For n = (N-1)

When (N-(N-1)) persons departed that is only 1 person staying in the system out of N persons.

$$f(x)=\frac{e^{-\mu x}\left(\mu x\right)^{\left[N-\left(N-1\right)\right]}\mu}{\left[N-\left(N-1\right)\right]!}$$

$f(x)\geq 0\forall x$ and $\int_{-\infty}^{\infty}\frac{e^{-\mu x}\left(\mu x\right)\left[{}^{N-\left(N-1\right)}\mu\right]}{\left[N\left(N-1\right)!\right]}$ dx

$$\int_{0}^{\infty}\frac{e^{-\mu t}\left(\mu t\right)\mu}{1!}$$

u = μ t   v= $e^{-\mu t}$

d μ = μ dt $\int v dv = \frac{e^{-\mu t}}{-\mu}$

$$\int_0^\infty e^{-\mu t}(\mu t)dt = \mu\left[\frac{\mu t\ e^{-\mu t}}{-\mu} - \int \frac{e^{-\mu t}}{-\mu}\mu\ dt\right]_0^\infty$$

$$=\left[-te^{-\mu t} + \frac{e^{-\mu t}}{-\mu}\right]_0^\infty = -\mu\left[e^{-\infty} + \frac{e^0}{-\mu}\right]$$

$$\int_0^\infty \mu e^{-\mu x}\mu x\ dx = 1$$

f(x) is density function for chosen random variable.

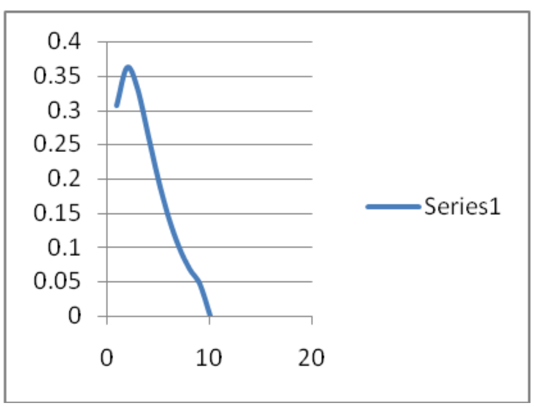

FIGURE - 1

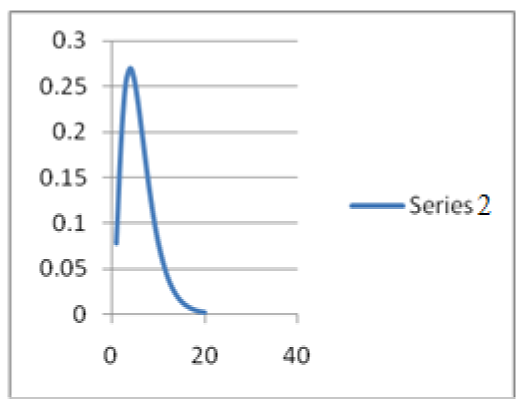

FIGURE - 2

Series-1 represents the probability density function graph of new random variable when n = 0.

X- represents time, Y- represents f(x).

Series-2 represents the probability density function graph of new random variable when n = 1.

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For n = (N-2)

When (N-(N-2)) persons departed, then there are only 2 persons in the domain restricted queueing system.

$$\int_0^\infty f(x)dx = \int_0^\infty \frac{e^{-\mu x}(\mu x)^{[N-(N-2)]}\mu}{[N-(N-2)!]}\,dx$$

$f(x) \geq 0 \; \forall x$ and $\int_0 f(x)dx = \int_0^\infty \frac{e^{-\mu x}(\mu x)^{[N-(N-2)]}\mu}{(N-(N-2))!}dx$

$$\int_0^\infty \frac{e^{-\mu t}(\mu t)^2 \mu}{2!}dt$$

$$\frac{\mu^3}{2}\left[\frac{e^{-\mu t}t^2}{-\mu} - \frac{e^{-\mu t}}{-\mu}2tdt\right]_0^\infty$$

$$\mu\left[\frac{e^{-\mu t}t^2}{-\mu} + \left[\frac{e^{-\mu t}t}{-\mu^2} - \int \frac{e^{-\mu t}}{-\mu}\right]\right]_0^\infty$$

$$\mu\left[\frac{e^{-\mu t}t^2}{-\mu} - \frac{e^{-\mu t}t}{-\mu^2} - \int \frac{e^{-\mu t}}{-\mu}\right]_0^\infty$$

$$\frac{e^{-\infty}e^0}{-1} = 1$$

$$\int_0^\infty \frac{e^{-\mu t}(\mu t)^2 \mu}{2!} = 1$$

$\therefore$ For chosen random variable f(x) is the density function.

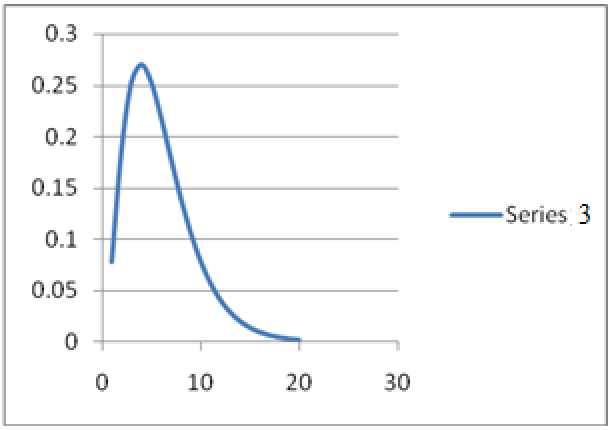

FIGURE - 3

Series 3 in figure-3 represents the probability density function graph of new random variable when n = 2.

∴ In general, for the chosen random variable departing only (N-n) persons from the system, the probability density function is.

$$f(x) = \begin{cases} \dfrac{e^{-\mu x}(\mu x)^{N-n}\mu}{(N-n)!} & for\ n \geq 0 \\ 0 & otherwise \end{cases}$$

This is the empirical probability distribution obtained from the estimation methods.

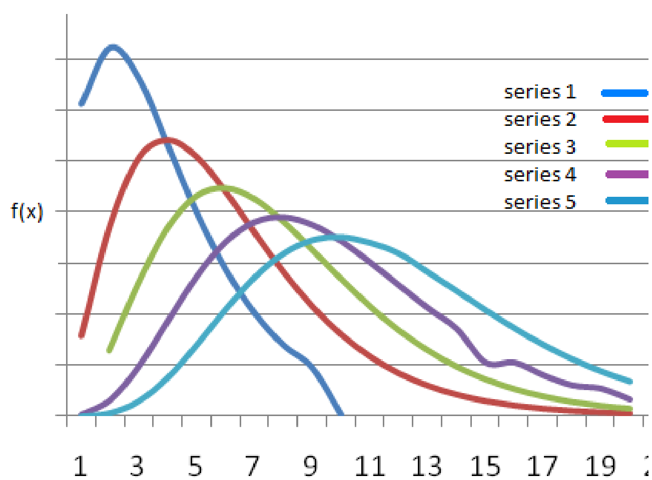

FIGURE - 4

Series 1 represents the probability density function graph when n = 0.

Series 2 represents the probability density function graph when n = 1.

Series 3 represents the probability density function graph when n = 2.

Series 4 represents the probability density function graph when n = 3.

Series 5 represents the probability density function graph when n = 4.

X- axis represents time, Y- represents f(x).

CONCLUSION

For the assumed a continuous random variable "how likely the interval to have 'N-n' fixed no. of departures out of N arrivals", the density function proposed is

$$f(x) = \begin{cases} \dfrac{e^{-\lambda x} (\lambda x)^n \lambda}{n!} & when \;\; x \geq 0 \\ 0 & otherwise \end{cases}.$$